Analysis and bifurcations of a dc-dc buck converter controlled by sine wave

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This thesis is dedicated to my parents for their love, endless support, encouragement and because they always believed in me. To my sisters who filled me up with a wonderful fraternal love. And to my girlfriend Kathe who every day inspired me to do my best and to deal with the obstacles.
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José Daniel Morcillo Bastidas

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Abstract

Low power systems are widely used in robotics and industrial areas; therefore, modeling and systems analysis provide reliable and best designs of such systems. The Buck converter is one of the systems that has been widely analyzed for years in order to understand its behavior and design better control algorithms. Accordingly, in this work the PWM voltage–controlled buck converter is modeled, simulated and studied. However, since many works have been developed under the study of the system controlled by ramp, this thesis will be mainly focused in the new study of the buck converter controlled by sine waveform (changing the non–smooth $T$–periodic ramp signal by the smooth $T$–periodic sine signal) operating in continuous and discontinuous conduction mode, converging towards the bifurcation analysis of almost all the parameters that govern the system. Additionally, some results obtained from both systems are compared.

Using a method for detecting events, the systems are simulated describing numerically all the phenomena found computing temporal responses, phase portraits, one dimensional, two dimensional and three dimensional bifurcation diagrams for different parameters. With this, every complex behavior will be easily recognized when a specific parameter is varied, not to mention new phenomena of the non–smooth nature are observed and described. Then, as bifurcation diagrams are computed varying the parameters ascending and descending, coexisting attractors, which are normally an undesired behavior in nonlinear systems, are observed and studied. Nevertheless, it is demonstrated that these bifurcation diagrams are not the sufficient remedy to find coexistence of solutions.

Finally, control algorithms are applied to the PWM voltage–controlled buck converter, which is a technique based on an adaptive control, where the $T$–periodic ramp signal ($V_{ramp}$) or the $T$–periodic sine signal ($V_s$) are modified to behave according to the control signal ($V_{co}$) and the input voltage ($V_{in}$) changes; all in order to extend even more the $V_{in}$ range over which the $1T$–periodic orbit remains stable. Moreover, with this control technique it is greatly reduced the percentage of regulation error ($\%e$) as well as it is eliminated the chaotic behavior when $V_{in}$ is varied. After all, numerical and experimental results, which show high agreement, are obtained so as to validate the performance of the system controlled by adaptive ramp.

Keywords: buck converter, PWM voltage–controlled, bifurcations, chaos, coexistence of solutions, chaos control.
**Resumen**

Los sistemas de baja potencia se utilizan ampliamente en la robótica y en la industria, por lo tanto, el modelado y análisis de sistemas proporcionan fiabilidad y mejores diseños para tales sistemas. El convertidor buck es uno de los sistemas que ha sido ampliamente analizado durante años con el objetivo de conocer su comportamiento y mejorar el diseño de los algoritmos de control. En consecuencia, en este trabajo el convertidor buck controlado por PWM de voltaje es modelado, simulado y estudiado. Sin embargo, dado que muchos trabajos se han desarrollado en el marco del estudio del sistema controlado por rampa, esta tesis se centra principalmente en el nuevo estudio del convertidor buck controlado por onda sinusoidal (cambiando la señal rampa $T$–periódica no suave por la señal sinusoidal $T$–periódica suave) operando en modo de conducción continua y discontinua, convergiendo hacia el análisis bifurcacional de casi todos los parámetros que rigen el sistema. Además, algunos resultados obtenidos a partir de ambos sistemas son comparados.

Utilizando un método para la detección de eventos, los sistemas son simulados describiendo numéricamente todos los fenómenos encontrados al computar respuestas temporales, retratos de fase, diagramas de bifurcaciones uno dimensionales, dos dimensionales y tridimensionales para diferentes parámetros. Con esto, todos los comportamientos complejos serán fácilmente reconocidos cuando un parámetro específico es variado, sin mencionar que nuevos fenómenos de la naturaleza no suave son observados y descritos. Posteriormente, ya que los diagramas de bifurcaciones son calculados variando los parámetros de forma ascendente y descendente, coexistencia de atractores, los cuales son normalmente un comportamiento no deseado en los sistemas no lineales, son observados y estudiados. No obstante, también se demuestra que estos diagramas de bifurcaciones no son el remedio suficiente para encontrar coexistencia de soluciones.

Finalmente, algoritmos de control son aplicados al convertidor buck controlado por PWM de voltaje, los cuales son una técnica basada en un control adaptativo, donde se modifica la señal rampa $T$–periódica ($V_{ramp}$) o la señal seno $T$–periódica ($V_s$) de tal forma que se comporten similar a la señal de control ($V_{co}$) y a los cambios en el voltaje de entrada ($V_{in}$); todo a fin de ampliar aún más el rango de $V_{in}$ sobre el cual la órbita $1T$–periódica se mantiene estable. Además, con esta técnica de control se reduce considerablemente el porcentaje de error de regulación ($\%e$) así como también se elimina el comportamiento caótico cuando $V_{in}$ es variado. Al final, resultados numéricos y experimentales, los cuales concuerdan altamente, son obtenidos con el fin de validar el funcionamiento del sistema controlado por rampa adaptativa.

**Palabras clave:** Convertidor buck, control por PWM de voltaje, bifurcaciones, caos, coexistencia de soluciones, control de caos.
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Symbols with Latin letters

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<th>Symbols</th>
<th>Term</th>
<th>IS Unit</th>
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<tr>
<td>C</td>
<td>Capacitance</td>
<td>$F$</td>
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<tr>
<td>D</td>
<td>Diode</td>
<td>$1$</td>
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<tr>
<td>e</td>
<td>Percentage error of the output voltage</td>
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<td>I</td>
<td>Current</td>
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<td>T</td>
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<td>u</td>
<td>Control signal</td>
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<td>V</td>
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Subscripts

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### Subscript Term

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<td>u</td>
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### Abbreviations

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<tr>
<td>DC</td>
<td>Direct current</td>
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<tr>
<td>PWM</td>
<td>Pulse width modulation</td>
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<tr>
<td>CCM</td>
<td>Continuous conduction mode</td>
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<tr>
<td>DCM</td>
<td>Discontinuous conduction mode</td>
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1 Introduction

Switched converters, due to its simplicity and efficiency, are widely used in a variety of applications, therefore there are currently few electronic devices which have no switched converters in its supply stage. The demand for a better performance and a less waste of energy in electronic devices causes that switched converters are subjected to increasingly stringent requirements. In practice, it is desired that switched power converters have less elements as possible; but this is not always possible since it is necessary to introduce elements of control and protection. In power converters is necessary a control stage because in general some of their components undergo changes over the time. However, the design of controllers for converters is not a trivial problem.

Nonlinear systems have been an important object of study considering that they are used to model switched circuits such as power converters, which are electronic devices extensively used to adjust voltage levels needed to supply several electronic equipment. The most used are the DC-DC converters, which are configurations of power electronics that allow, from a constant DC source, to control the DC voltage in the converter output [36], [15], [35]. Some of the most important applications are: sources of energy in computers, distributed power systems, power systems in electric vehicles and so on. Therefore, due to the applicability of such systems detailed studies become relevant. As a consequence and given the importance of studying power converters, throughout this thesis a dynamical analysis of the buck converter, controlled by sinusoidal and ramp signals, is accomplished.

Model dynamical systems have also played for years an important role in power converters, and model electrical circuits have contributed to improve the operation designs specially in switching circuits. The first investigation in this topic is presented in [12], where there was numerical, analytical and experimental results in the study of a PWM voltage–controlled buck converter, studies that in this thesis are similarly presented with the same converter and then improved with the buck converter controlled by sine waveform, which means to use a smooth $T$–periodic signal instead of a non–smooth one.

With the non–smooth nature the engineers community awakened interest in the discussion of switching systems as they have large area of applications and non–linear richness, as a result many methods of analysis have been gradually discovered such as iterated mappings, a nonlinear discrete modeling technique that was developed to describe dc–dc converters in
[21], or in [16], where the chaotic behavior in the buck converter was described. Likewise, the present work reexamines some of the studies previously mentioned and makes deep and new research in the buck converter controlled by sine waveform, focusing on the analysis of complex behaviors. Besides, many investigations developed in [4, 6, 8, 43, 13, 39] helped to elaborate this thesis, where analysis of period doubling route to chaos, subharmonics, skipping, sliding, grazing and border collision bifurcations were disclosed in the complex behavior exhibited by switched mode dc–dc converters.

Sudden jumps from periodic solutions to chaos, from chaos to periodic solutions, from periodic solutions to higher or lower periodic solutions, or just sudden jumps of the main attractor are transitions caused by border collision or grazing bifurcations and only seen in nonlinear dynamical systems; some of these phenomena were explained in [39, 5, 43, 8] but also reintroduced in this work with similar complex behaviors as tangent bifurcations, which are observed in the buck converter controlled by sine waveform.

Chaos is a random behavior which is normally undesired and appears in most of the power converters, thereby chaos has been deeply studied and used due to its noise–like behavior; therefore, in [20] applications of chaos are considered because, as we shall see below, the chaotic behavior emerges easily although control techniques are tested [33, 3].

Control of low power systems is a topic of concern in the industry due to still exist dynamic problems in some devices, problems that can be solved understanding the complex behavior of the systems. Modeling and analyzing dynamical systems requires various techniques of study, and in the past, many methods to model dynamical systems, specially methods to model power converters were introduced ([31, 21, 8]) with the aim of obtaining an approximation of the systems dynamic. But over the time new forms of study have found that there exists almost an infinite complex dynamic [39], because of that, tools like Poincaré map, bifurcation diagrams of codimension one and two, Lyapunov exponents, domains of attraction, among others, have helped to discover and understand the strange behavior exhibited by the nonlinear systems as well as contributed to the development of this thesis. An important review of the aforementioned nonlinear phenomena, different modeling approaches and the main results found in the last years can be found in [14], where the buck and boost converters were the main circuits of study.

One of the important strategies of analysis in power converters is basins of attraction, this tool helps to determine the systems sensitivity to initial conditions and ensure that the systems exhibit a desired behavior taking initial conditions from an adequate region. Coexistence of solutions are principally studied in [4], where the authors reported coexisting attractors with fractal basin boundaries in the voltage-mode controlled buck converter, and in [7], where is revealed the dynamical richness of non-smooth systems with tools like bifur-
cation diagrams, invariant manifolds, basins of attraction, and so on; all the results reported were obtained for a DC-DC buck converter controlled by PMW using a ramp waveform as $T$–periodic signal. Furthermore, to observe and analyze different responses and phenomena, in this thesis we also investigate the presence of coexisting attractors in the buck converter periodically forced by a $T$–periodic sine waveform.

Many researchers have focused their attention to the study of control techniques for switching power supplies, specially the buck converter which is one of the most common converters used in a variety of applications [34, 37]. Lately the aim of engineer community is to improve efficiency and size of the buck converter increasing switching commutation to reduce the inductor size [28]. In fact, in the last decade several strategies have been proposed for controlling the buck converter in order to overcome aforementioned features [11, 16, 3]. Nevertheless these strategies exhibit a wide regime of chaos and a plethora of complex behaviors. These behaviors strongly affect the buck converter performance and exist many techniques developed to counteract chaotic regimes and another undesired behaviors [33, 9, 18, 45, 19]. The most used chaos control techniques have been sliding mode control [22, 17] and FPIC strategy [1, 2], techniques that have been developed for some authors to provide robust and efficient responses. In order to eliminate the orbits of period greater than one and chaos behavior, mainly OGY (proposed by Ott, Grebogi and Yorke [32]) and TDAS (proposed by Pyragas [23]) were proposed. The first method is basically based on introducing a small perturbation to an accessible parameter when the system operates in an undesired behavior, and the second method involves a control signal and the period of the orbit, using delayed samples of the states variables; it is called auto synchronization. Both methods eliminate chaos and orbits of period greater than one successfully in the buck converter and an example can be found in [14]. However, these techniques require complex schemes to be implemented as they need digital devices such as FPGAS, DSPs, microcontrollers and so on. In addition, as there are many different chaos control techniques that provide several and different advantages, we look at part of them for general comments. For instance, in [33, 19, 45, 18] can be found the outstanding references in chaos control applied to power converters. Fundamentally, some of these techniques take advantage of the existence of unstable periodic orbits in the strange attractors; one way is applying small perturbations to push the state toward the stable manifold; the other one is targeting the unstable fixed points in every cycle or applying small changes in the switching-on instants in every cycle as well. Others use stability analysis and Floquet multipliers to find out how to stabilize the unstable periodic orbits. Also, perturbing $V_{ref}$ in the system equations or improving any control technique adding an extra discovery. With that, these techniques exhibit features such as shorter or longer settling time, smaller or larger steady state errors, more or less oscillations and slower or faster responses.

On the other hand, Jonathan H.B. Deane and David C. Hamill proposed a basic voltage-mode control applied to a DC-DC buck converter based on a PWM using a simple ramp
(with fixed slope and fixed offset voltage) as a $T$-periodic signal [12]. This simple technique makes the system regulate for some $V_{in}$ values, but appear orbits of period greater than one and chaotic bands when $V_{in}$ varies, and in real applications where $V_{in}$ could vary to external changes it decreases the system performance.

Interested in developing an easier control method and simple to implement, in this thesis it is also proposed a novel control scheme which suppresses the chaotic bands and the orbits of period grater than one, in a wider range of $V_{in}$ for the buck power converter controlled by ramp. Essentially, this method that we have called “adaptive–ramp control”, consist on adapting the ramp waveform ($V_{ramp}$) to the control signal ($V_{co}$) in such a way $V_{ramp}$ becomes similar to $V_{co}$. In other words, we have assumed that the ramp signal can change its slope and offset voltage over the time. As a result, with our proposed technique we have mainly proven numerically that the $1T$-periodic orbit remains stable within the $V_{in}$ range $[13, 70]V$, and we have also validated experimentally for the $V_{in}$ range $[20, 40]V$ using a ramp with fixed slope but variable offset voltage.

In the past, similar techniques have been proposed, some papers discuss the concept of modifying the ramp features in PWM dc-dc converters, aiming at improve the performance of wide operation range. Firstly, it is discussed about changing the amplitude in [10, 25, 24]. The method is called “$V_{in}$ feed–forward”, and it is based on designing the ramp amplitude to be proportional to $V_{in}$ values. Afterwards, the concept of a variable slope is discussed in [40, 42, 41], where it can be found different ways to automatically adjust the ramp slope so as to avoid output variations or subharmonic oscillations. Yet, non of them provide bifurcation analysis to observe clearly the performance improvement in a wider operation range of $V_{in}$, and it is not considered a variable offset voltage.

Summarizing, in the present thesis it is firstly modeled the buck converter controlled by ramp and sine waveform. The simulation method is described in detail and how to identified non–smooth bifurcation in both converters is also explained. Secondly, the buck converter controlled by ramp and sine waveform are simulated to observe the accuracy of the simulation method proposed in the first chapter as well as to validate that the systems were rightly modeled to work in $CCM$ and $DCM$. Thirdly, one–dimensional bifurcation diagrams are computed and analyzed in both converters for the most important parameters. Fourthly, basins of attraction diagrams are obtained and studied in the buck converter controlled by sine waveform once coexistence of solutions were evidenced in the bifurcation diagrams. And finally, control algorithms are proposed in order to counteract chaotic motion when $Vin$ is varied in both converters under analysis.

On the other hand, during the development of this project, it was obtained the following scientific production:

- “Análisis y Bifurcaciones de un Convertidor Buck DC-DC Controlado por onda Senoidal”, José Morcillo and Gerard Olivar. School of Applied Mathematics and Innovation
(SAMI 2010), Non-smooth systems and applications. Universidad Sergio Arboleda, Santa Marta, Colombia, 2010. Accepted.


- Presentation and socialization of the main results in The Hong Kong Polytechnic University and The City University of Hong Kong.
2 General Conceptual Framework

Abstract

This chapter presents the basic theory to understand the development of this work. First it is defined the basic system case of study and its corresponding mathematical model. Subsequently, it is shown the control scheme that has been used to make the systems regulate which is divided into two parts: using a ramp signal (scheme widely studied) and using a sine waveform, both as periodic signals responsible for the control signal generation or PWM. Continuing, it is also proposed a simulation method using events to reproduce the buck converter response in closed loop. Finally, a classification of non–smooth bifurcations for both systems under analysis is shown.
2.1 The buck converter

A circuit well known and used in power electronics is the Buck converter, whose function is to convert a dc voltage to a lower dc voltage. As can be seen in Figure 2-1, the circuit is a second order system due to its two states variables, $L$ (inductor) and $C$ (capacitor); $R$ is the load resistance and $Vin$ is the voltage source. We assume that all the components in the circuit are ideal, so when $S1$ turns ON and OFF it has zero and infinite resistance respectively. The switch $S2$ is uncontrolled and it is normally replaced by a diode, but the switch $S1$ is frequently controlled by a PWM (Pulse–Width Modulation).

![Figure 2-1: Buck converter](image)

Throughout this thesis we consider the circuit working in continuous and discontinuous conduction mode, $CCM$ (since the inductor passes current without a break) and $DCM$ (the inductor current can become zero for part of the cycle as the diode comes out of conduction) respectively. Besides, in order to model the Buck converter in a comfortable manner, it is divided into three topologies. Figure 2-2 shows the different circuit topologies according to the switches ($S1$ and $S2$) position.

![Figure 2-2: Buck Topologies](image)

In topology 1 the input provides energy to the inductor and load, and considering that $S2$ is replaced by a diode, it is reverse biased. In topology 2 the inductor current flows through the diode which is forward biased, transferring some of its stored energy to the load. And in
topology 3, because $I(t)$ and $S1$ are zero, the diode is reverse biased and the energy stored in the capacitor is discharged through the load.

### 2.1.1 ODEs that model the converter

Each topology has its own linear differential equations that can be derived by the standard Kirchoff’s laws:

**Topology 1**, see Figure 2.2(a).

$$
\frac{d}{dt} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} = \begin{pmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{pmatrix} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} + \begin{pmatrix} 0 \\ V_{in}/L \end{pmatrix} \tag{2-1}
$$

**Topology 2**, see Figure 2.2(b).

$$
\frac{d}{dt} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} = \begin{pmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{pmatrix} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} \tag{2-2}
$$

**Topology 3**, see Figure 2.2(c).

$$
\frac{d}{dt} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} = \begin{pmatrix} -1/(RC) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} \tag{2-3}
$$

When the system works with topologies 1 and 2 or with Eqs. (2-1) and (2-2), operates in CCM, and the system operates in DCM when works with topology 3 or equation (2-3). Moreover, the system is non–autonomous due to switching condition is time dependent.

Figure 2-3 shows the equilibrium point of each topology using the component values: $L = 20mH, C = 47\mu F, R = 22\Omega, V_{in} = 20V$. The equilibrium points are given (as in [31]) by:

**Case topology 1**: a stable focus at $(V_{in}, V_{in}/R) = (20V, 0.9090A)$.

**Case topology 2**: a stable focus at (0V,0A).

**Case topology 3**: a stable focus at (0V,0A).

It can be seen in Figure 2.3(a) the system solution of topology 1, every point within the phase diagram tends to the stable focus whose value is ($20V, 0.9090A$). For this topology the initial conditions were taken as $V_C(0) = 0V$ and $I_L(0) = 0A$ because the input voltage ($V_{in}$) provides energy to the components causing an increased in the output voltage ($V_C(t)$). Figure 2.3(b) shows the phase diagram of topology 2, where the trajectory of the system solution and every point around tend to the origin (0V,0A). In this case the equilibrium point of topology 1 was taken as initial condition ($V(0) = 20V, I(0) = 0.9090A$), this due to
the absence of the input voltage. And in Figure 2.3(c) (where the initial condition was also taken as the equilibrium point of topology 1) is shown the immediate change of the current \((I(t))\) from 0.9090\,A to 0\,A and the exponential decrease of the voltage \((V(t))\), which in this figure is like a straight line because the capacitor discharge time is small. In [29] the equilibrium points of topologies 1 and 2 and their stability were studied; namely, the system working in \(CCM\) and not considering \(DCM\) because they assumed that the system had bidirectional switches, which allow negative currents.

2.2 Buck converter controlled by ramp

The buck converter controlled by \(PWM\) is formed through a feedback loop normally called voltage-mode control, which consists on a control signal \((V_{co})\) that is assumed to be the error between a reference voltage \((V_{ref})\) and the measured capacitor voltage \((V(t))\), then
$V_{co}$ is compared with a T-periodic sawtooth waveform. So the control pulses which drive $S_1$ or $u$ (See Figure 2-1) switch whenever the two signals become equal. Figure 2-4 shows the situation exposed above, where $A_1$ is an OPAM that has gain $a$; according to this, we can write:

$$V_{co}(t) = a(V_C(t) - V_{ref})$$

(2-4)

![Figure 2-4: Closed loop scheme taking a sawtooth as a T-periodic waveform.]

Now, considering that $A_2$ is a comparator we can define:

$$u(t) = \begin{cases} 
1 & \text{if } V_{co}(t) < V_{ramp} \\ 
0 & \text{if } V_{co}(t) > V_{ramp} 
\end{cases}$$

(2-5)

Eq. (2-5) indicates the conditions for which $u$ commutes between 0 or 1. The above definitions do not include DCM, but in that case where $I(t) = 0A$, simply $u(t) = 0$.

The ramp waveform in Figure 2-4 is given by:

$$V_{ramp}(t) = V_{Lo} + (V_u - V_{Lo}) \frac{t}{T}$$

(2-6)
2.3 Buck converter controlled by sine wave

Here, $V_{Lo}$ and $Vu$ in Eq. (2-6) is the lower and upper voltages of the ramp waveform, and $T$ their period.

2.3 Buck converter controlled by sine wave

The non-smooth T-periodic sawtooth waveform in the buck converter controlled by PWM can be changed for a smooth T-periodic sine wave, this because the first one has been well studied and also in order to discover interesting behaviors analyzing the buck converter response using a smooth signal instead of a non-smooth one.

![Figure 2-5: Closed loop scheme taking a sine wave as a T–periodic waveform.](image)

Figure 2-5 shows the situation exposed above. The control scheme of this system is the same proposed in Section 2.2, but the T-periodic signal which in the present case is a sine wave is given according to the same properties of the ramp signal by:

$$V_s(t) = V_{Lo} + \left(\frac{Vu - V_{Lo}}{2}\right) \left(1 + \sin\left(\frac{2\pi t}{T}\right)\right)$$  \hspace{1cm} (2-7)
thus, we can define the switching signal as follows:

\[
    u(t) = \begin{cases} 
        1 & \text{if } V_{co}(t) < V_s \\
        0 & \text{if } V_{co}(t) > V_s 
    \end{cases}
\]  

(2-8)

2.4 How to simulate the converters through events

Many dynamical systems exhibit a non-linear behavior in some cases due to discontinuities, switchings, impacts or any kind of surfaces where the system suffers a sudden change. In this work the aforementioned surfaces or conditions in which a system undergoes discontinuity are going to be treated as an event. This method for modeling dynamical systems is based on obtaining the ordinary differential equations (ODEs) that describe the system and implement them in the tool called ODE of MATLAB (we have used ODE45 for our systems); this tool or function has the option of introduce the events, it means that inside of the function is defined the system restrictions which determine the non-linear surface where the system must change its behavior; namely, the tool ODE45 is going to integrate the ODEs until finds the restriction, or the event, where is necessary to change the topology, equations or the system behavior; then, it continues integrating until finds the next event or restriction, and in that way the system changes its topology as an event occurs.

To adequately model the systems under study, with the company of events detection is also necessary to draw a states transition diagram, where we can connect the events or the states through rules that allow the change from one state or event to another; hence, working together, the events detection and the states transition diagram is achieved a very precise method for modeling dynamical systems, a method that has demonstrated good results which will be observed later simulating the two systems comprised in our study, the buck converter controlled by ramp and sine waveforms.

To simulate our systems in closed loop and described them adequately according to the proposed simulation method, it is necessary to take into account the following.

We first need to declare the events:

- When $V_{ramp}$ (or $V_s$) and $V_{co}$ become equal.
- When $I(t)$ reaches zero.
- And when $t$ reaches $nT$ (where $n=1,2,3,...$). This in order to sample the state variables of the systems during each period.
2.5 Identifying non-smooth bifurcations

Once the events are declared, at the same time is necessary to draw a states transition diagram because the different topologies have to be connected according to the switching signal \((u)\) and the inductor current \((I(t))\). Figure 2-6 shows the diagram mentioned before. This diagram also has to be programmed in MATLAB.

![States transition diagram for the buck converter in closed loop.](image)

Each topology in Figure 2-6 indicates a state and each arrow indicates the states change according to the conditions or rules; the initial state is generally known, and in this case the initial conditions of the systems \((I(0), V(0))\) give the initial state.

2.5 Identifying non-smooth bifurcations

2.5.1 System controlled by ramp

Identifying the occurrence of non-smooth bifurcations has been an increasingly topic of analysis. Here we are going to define some terms that will help us to classify the border collision episode in the buck converter controlled by ramp (see Figure 2-4), based on the analysis that some papers have published about this issue [26, 27, 44].

The system response tends to undergo complex behaviors when \(V_{co}\) touches or approaches to some specific regions of \(V_{ramp}\) (see Figure 2-7). Border collision bifurcations take place when \(V_{co}\) touches the lower and upper tip of \(V_{ramp}\), but also other non-smooth bifurcations could occur when \(V_{co}\) grazes other regions of \(V_{ramp}\). This situations is explained in Figure 2-8.
We have classified the non-smooth bifurcations for the system controlled by ramp according to the intersecting points of $V_{co}$ with $V_{ramp}$. In Figure 2-8 can be seen that any intersecting point of $V_{co}$ with the ramp slope has been named $C$, and when $V_{co}$ crosses the vertical edge of $V_{ramp}$ a $D$ point is defined. Depending on the $C$ and $D$ movements the bifurcations are classified as follows:

- As can be seen in Figure 2.8(a), when any parameter is been varied, both $C$ and $D$ move upward until become one point at the vertex of $V_{ramp}$, this type of border collision is called C-type. When the system experiences this phenomena can be seen that the switching action is disrupted, giving rise to a sudden jumps or leaps (from periodic orbits to chaos or from periodic orbits to periodic orbits and vice versa) in the bifurcation diagrams due to operational changes in the system.

- From Figure 2.8(b) can be noticed that only $D$ moves upward and creates a $C'$ point in the ramp slope as any parameter is varied, this type of border collision is named D-type. In this case, the durations of the on and off intervals are disrupted, however, the system operation is not affected since the switching action is maintained; as a result, an inflection in the bifurcation diagrams is produced. Nevertheless, if the system is very sensitive to changes in the duration of the pulses, the stability is lost.

- The grazing-type from Figure 2.8(c) is produced when $V_{co}$ is tangent to $V_{ramp}$, in this case the switching action is disrupted as well, therefore operational changes are caused and sudden leaps or lost of stability in the bifurcation diagrams are observed.

When $V_{co}$ grazes the maximum or minimum value of $V_{ramp}$ ($V_{u}$ or $V_{lo}$), or is tangent to $V_{ramp}$, the system could undergo chaotic or periodic behaviors, that depends on whether the system remains stable or unstable after cross the border (see Figure 2-9).
2.5 Identifying non-smooth bifurcations

(a) C-type.

(b) D-type.

(c) Grazing-type.

Figure 2-8: Classification of non-smooth bifurcations for the system controlled by ramp.

Figure 2-9: State space. Region A and B are smooth. In the Border the system equations are not differentiable.
If we consider Figure 2-9 as the state space of the system, and then we divide it into two regions, a border appears between the Regions A and B, where the border is supposed to be the upper and lower values of \( V_{ramp} \), or any point which \( V_{co} \) grazes. Therefore, we can say that the set of differential equations or the stroboscopic map of the system in the Border, unlike in Regions A or B (smooth), are not differentiable. Hence, such maps are called \textit{piecewise smooth}. For that reason, when a periodic orbit crosses the border as any parameter is varied, the orbit can become unstable; as a result, the system response bifurcates into chaotic or higher periodic orbits; such bifurcations are called \textit{border collision bifurcations}.

\subsection*{2.5.2 System controlled by sine waveform}

Identifying non–smooth bifurcations in this case has some differences comparing with the system controlled by ramp. Here we are also going to define, as we explained in the past subsection, some terms that will help us to classify the border collision episode or in this case also called tangent bifurcations for the buck converter controlled by sine waveform (see Figure 2-5).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{Switching logic of the buck converter controlled by sine waveform. \( V_{co}(t)-V_s(t) \).}
\end{figure}

The system response depends on the switching action and to observe a classic interaction between \( V_{co} \) and \( V_s \), which are the signals responsible of the switching action, in Figure 2-10 have been plotted the two signals.

Border collision bifurcations in this case are considered as tangent bifurcation due to the smoothness of the \( T \)-periodic sine waveform, it means that if \( V_{co} \) grazes \( V_s \), a bifurcation of non–smooth type occurs no matter which part of \( V_s \) touches \( V_{co} \). However, there are six regions which are mainly touched by \( V_{co} \). This can be seen in Figure 2-11.

From Figure 2-11 can be seen that \( V_{co} \) (red curves) can mainly graze six different regions in one period of \( V_s \) and produces non–smooth bifurcations. In Figure 2-12 has been explained
2.5 Identifying non-smooth bifurcations

Figure 2-11: Six possible regions that lead to a non-smooth bifurcations.

in detail the first three regions because, the regions 4,5 and 6 are basically the same situations than 1,2 and 3 respectively.

In Figure 2-12 has been classified the non-smooth bifurcations for the system controlled by sine waveform according to the more possible regions that \( Vco \) can graze in \( Vs \). It can be also seen the occurrence of each region with its respective switching signal. And from all of them, we can say that no matter which region \( Vco \) grazes, there will be always a disruption in the switching action, wherewith the system can undergo an operational change.

As we explained with the system controlled by ramp, each region can be considered as a non-smooth border (as in Figure 2-9), which means that if \( Vco \) crosses the border and becomes chaotic, it suggests that \( Vco \) underwent any type of the six grazing behaviors or a tangent bifurcation, and the disruption in the switching action caused a destabilization in the system giving rise to a chaotic behavior. Or it could be possible that \( Vco \) crosses the border and remains stable, which means that the tangent bifurcation would produce a leap or an inflection in the bifurcation diagram.

2.5.3 Non-smooth bifurcations of current-type

Both the buck converter controlled by sine waveform and the buck converter controlled by ramp undergo non-smooth bifurcations of the current-type. In Figure 2-13 can be observed how is produced a current-type bifurcation.

As can be observed from Figure 2-13, phase portraits are plotted indicating the time durations and the respective topologies that are working during each time interval. The number one means that the Topology 1 (see Figure 2.2(a)) is working from \( t1 \) to \( t2 \), the number two means that the Topology 2 (see Figure 2.2(b)) is working from \( t2 \) to \( t3 \), and the number three means that the Topology 3 (see Figure 2.2(c)) is working from \( t3 \) to \( t1 \). This is also represented for each phase portrait with the switching actions.
Figure 2-12: Classification of non-smooth bifurcations for the system controlled by sine waveform.
As can be analyzed, this type of non-smooth bifurcation takes place not only due to changes in the switching durations but also due to operational changes. In other words, once the inductor current leaves the zero, $t_3$ disappears and the Topology 3 does not work any more; as a consequence, the on and off durations are altered and the system only enters to work with the Topology 1 and 2, which means an operational change. In this case, the zero can be considered as a border as well; thus an orbit can cross the border and bifurcates into chaos, orbits of period greater than one or just remains stable, but always altering a feature in the bifurcation diagram.

**2.6 Conclusions**

- It was found all the topologies that integrate the buck converter, then was explained the conditions that lead the system to each of them.

- It was modeled each of the topologies mentioned in the previous item using ODE’s. It was also plotted the equilibrium points of each of the topologies.

- It was illustrated the control law of the two different systems under analysis, the buck converter controlled by ramp and the buck converter controlled by sine wave, explaining the differences between them and showing their respective circuit diagram.

- It was proposed a simulation method through events, aiming at getting accuracy, reliability and easy way to simulate dynamical systems.

- Non-smooth bifurcations for the system controlled by ramp were classified. Moreover, it was described the resulting changes in the system behavior under border collision and grazing bifurcations, all appearing due to disruption in the pulses duration and operational changes.
• Non-smooth bifurcations for the system controlled by sine waveform were classified too. It was showed that any tangent bifurcation produced a disruption in the pulses duration or a change in the system operation, as a result, sudden leaps, lost of stability or inflections in the bifurcation diagrams are observed.

• A current-type bifurcation was described. This type of bifurcation can occur in both the buck converter controlled by ramp and in the buck converter controlled by sine waveform. Besides, this bifurcation produces the same effects in the bifurcation diagrams than the border collision bifurcations in the buck converter controlled by ramp.
3 Simulation of the Buck Converter

Abstract
In this chapter is tested and evidenced that the simulation method proposed in the previous section works perfectly; firstly simulating the buck converter controlled by ramp taking the same parameter values reported in some references with the aim of comparing the accuracy and reliability of the method. Then, simulating the buck converter controlled by ramp and the buck converter controlled by sine wave with different component values, it is pointed out that the systems were modeled to work in CCM and DCM as well as both regulate and present diverse behaviors.
3.1 Simulation of the buck converter controlled by ramp

Since the topologies and conditions of the buck converter have been modeled in an adequately order from Section 2.1 to Section 2.4, first it is simulated the circuit of Figure 2-4, using Eq. (2-6) as the T-periodic waveform is a ramp. Table 3-1 shows the component values used in the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>20mH</td>
<td>Vref</td>
<td>11.3V</td>
<td>a</td>
<td>8.4</td>
</tr>
<tr>
<td>C</td>
<td>47µF</td>
<td>VLo</td>
<td>3.8V</td>
<td>T</td>
<td>400µs</td>
</tr>
<tr>
<td>R</td>
<td>22Ω</td>
<td>Vu</td>
<td>8.2V</td>
<td>Vin</td>
<td>35V</td>
</tr>
</tbody>
</table>

Table 3-1: Table of component values to simulate the buck converter controlled by ramp.

(a) Vco and Vramp are simulated including the switching signal (u). 5 periods (5T) are plotted.

(b) i_L-V_C phase portrait. 5T are plotted.

Figure 3-1: Numerical simulations corresponding to Figure 2-4 circuit.

It has been obtained in Figure 3-1 the ramp and control voltage waveforms (Vramp and Vco) with the switching signal and the phase portrait using the values of Table 3-1, which have been defined according to [12], aiming at comparing the results and verifying the accuracy of our simulation method. Now, in Figure 3-1 can be seen that the diagrams are really similar to those shown in [12], where there was good agreement with the experimental results, in
3.1 Simulation of the buck converter controlled by ramp

[16], where there was an analysis of chaos for the Figure 2-4 circuit, among others, in [7], or [21]. The diagrams of Figure 3-1 are simulations of the system undergoing chaotic behavior, but they can be seen better with more transients in Figure 3-2, where is shown the phase plane and the stroboscopic map for the chaotic attractor exhibited by the buck converter controlled by ramp in the chaotic zone, and as can be seen, all the diagrams are equal to those obtained in the references aforementioned, which means that our method is truly accurate.

![Diagram](image)

(a) $i_L-V_C$ phase plane. 1000T are plotted. (b) Strange attractor obtained with 75000 iterations.

**Figure 3-2:** Numerical simulations of the chaotic attractor and the phase plane for $Vin = 35V$.

**The A-switching map strategy**

In [8] was introduced a new discrete time non-linear map called **A-switching map**, it was proposed to analyze dc/dc converters and specially used to obtain and study the bifurcation diagram taking $Vin$ as bifurcation parameter, and the chaotic attractor for $Vin = 35V$ in the circuit of Figure 2-4; therefore, in order to show the fidelity of our simulation method they were also built and plotted in Figure 3-3. As a result, from those diagrams can be deduced that the **A-switching map** can also be numerically calculated by events.

**3.1.1 System working in CCM**

As it was seen in the previous section, the system was undergoing a chaotic behavior and also was working in CCM; so we chose that specially behavior because we wanted to probe how our simulation method works properly. Having seen that, right now we want to show
how the system regulates changing $Vin$ and also show specifically which operation mode is experiencing the system.

Taking the component values of Table 3-1 and changing $Vin = 20V$, it is obtained the buck converter response in Figure 3-4. And as can be seen, the system regulates in a 1T-periodic orbit, following perfectly the reference ($V_{ref}$). Moreover, from Figure 3.4(b) and 3.4(c) can be observed that the inductor current $i_L$ never drops to zero, so we conclude that the system definitely is working in $CCM$, it means that the system is commuting between topologies 1 and 2 (See Figure 2-2).

### 3.1.2 System working in DCM

So as to see whether the buck converter controlled by ramp has also been modeled to work in $DCM$, it has been simulated the system taking the component values of Table 3-1 but changing $Vin = 20V$, $R = 100\Omega$ and $L = 3.5mH$, the simulation results can be seen in Figure 3-5.

As can be observed from Figure 3.5(b) and 3.5(c), the inductor current $i_L$ drops to zero part of the cycle, suggesting that the system is working with topologies 1, 2 and 3; concluding so our system has been modeled to work in $DCM$ suitably. Also, it can be noted that the system response is presenting a 1T-periodic orbit from Figure 3.5(d); In addition, since the system was correctly modeled to work in $DCM$, the system can regulate to follow the reference ($V_{ref}$) as can be seen in Figure 3.5(a).
3.2 Simulation of the buck converter controlled by sine wave

(a) Capacitor voltage “$V_C$” in the time domain.

(b) Inductor current “$i_L$” in the time domain.

(c) $i_L$-$V_L$ phase plane.

(d) $V_{ramp}$ and $V_{co}$ in the time domain plotted in steady state.

**Figure 3-4**: Numerical simulations of the buck converter controlled by ramp exhibiting a $1T$-periodic orbit for $V_{in} = 20V$. System working in CCM. 150T are plotted.

3.2 Simulation of the buck converter controlled by sine wave

As mentioned in Section 3.1, our proposed simulation method worked perfectly showing good accuracy with some results of some references. So knowing that our proposed simulation method is reliable and that the system has been modeled covering all its topologies, we are going to show the simulation of the system of Figure 2-5, the buck converter controlled by sine wave, working in $CCM$ and in $DCM$, but using Eq. (2-7) as the $T$-periodic waveform is a sine in this case.
Figure 3-5: Numerical simulations of the buck converter controlled by ramp exhibiting a $1T$-periodic orbit. System working in DCM.

3.2.1 System working in CCM

Firstly, it is going to be seen the system working in CCM, for that in Table 3-2 is shown the component values used to simulate the present system, assigned trying to use the same values reported in Table 3-1, this in order to compare in some features the buck converter controlled by ramp with the buck converter controlled by sine wave because, despite that some parameter values are not the same, both systems could present similar dynamics. The resulting diagrams can be seen in Figure 3-6.
3.2 Simulation of the buck converter controlled by sine wave

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
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<td>Vref</td>
<td>12V</td>
<td>a</td>
<td>8.4</td>
</tr>
<tr>
<td>C</td>
<td>47μF</td>
<td>V\text{Lo}</td>
<td>3.8V</td>
<td>T</td>
<td>400μs</td>
</tr>
<tr>
<td>R</td>
<td>22Ω</td>
<td>Vu</td>
<td>8.2V</td>
<td>Vin</td>
<td>20V</td>
</tr>
</tbody>
</table>

**Table 3-2:** Table of component values to simulate the buck converter controlled by sine wave working in CCM.

(a) “\(V_C\)” in the time domain.  
(b) “\(i_L\)” in the time domain. Undergoing CCM.  
(c) \(i_L-V_C\) phase plane.  
(d) \(V_s\) and \(V_{co}\) in the time domain.  

**Figure 3-6:** Numerical simulations of the buck converter controlled by sine wave exhibiting a 1\(T\)-periodic orbit. System working in CCM. 100\(T\) are plotted.
Figure 3.6(a) shows that “$V_C''$ follows $V_{ref}$, it means that the system is regulating properly; Besides, from Figure 3.6(b) can be noted that the system is clearly working in $CCM$ as “$i_L''$ never drops to zero, and as occurs with the buck controlled by ramp, this system is also commuting between topologies 1 and 2. Furthermore, Figure 3.6(c) and 3.6(d) show that the system provides a 1T-periodic orbit as is commonly desired.

### 3.2.2 System working in DCM

To find the system working in $DCM$ was necessary to change some parameter values, hence, in Table 3-3 are shown the component values used to obtain the simulation diagrams of Figure 3-7.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3.5mH</td>
<td>$V_{ref}$</td>
<td>12V</td>
<td>$a$</td>
<td>8.4</td>
</tr>
<tr>
<td>$C$</td>
<td>47µF</td>
<td>$V_{Lo}$</td>
<td>3.8V</td>
<td>$T$</td>
<td>400µs</td>
</tr>
<tr>
<td>$R$</td>
<td>100Ω</td>
<td>$V_{u}$</td>
<td>8.2V</td>
<td>$V_{in}$</td>
<td>20V</td>
</tr>
</tbody>
</table>

**Table 3-3**: Table of component values to simulate the buck converter controlled by sine wave working in DCM.

As we have explained before, from Figure 3.7(a) can be seen that the system regulates following $V_{ref}$. Likewise to check that the system is working in $DCM$ it can be observed Figure 3.7(b) and 3.7(c), where it can be clearly seen the system commuting between topologies 1, 2 and 3 as well “$i_L''$ drops to zero in part of the cycle; including that the system is going through 1T-periodic orbit as can be seen in Figure 3.7(c) and 3.7(d).

Moreover, Figure 3.7(c) shows the 1T-periodic orbit once transients have been eliminated, in other words, it can be appreciated the periodic orbit in steady state.

### 3.3 Conclusions

- Simulating the buck converter controlled by ramp and comparing the results with some references, it was proved that our simulation method performs flawlessly, showing that the complex behaviors obtained were really similar to those reported in some past research works.
3.3 Conclusions

(a) “$V_C$” in the time domain.

(b) “$i_L$” in the time domain. Undergoing DCM.

(c) $i_L$-$V_C$ phase plane.

(d) $V_s$ and $V_{co}$ in the time domain.

**Figure 3-7:** Numerical simulations of the buck converter controlled by sine wave exhibiting a 1T-periodic orbit. System working in DCM. 60T are plotted.

- It was probed that through events can be also executed the A-switching map strategy proposed in [8].

- It was shown through simulations that the systems, the buck converter controlled by ramp and the buck converter controlled by sine wave, were rightly modeled to work in CCM and in DCM.

- It could be seen through phase portraits, $V_{ramp}$ and $V_{co}$ signals how the systems present chaotic behaviors and 1T-periodic orbits as some component values were changed.

- From phase portraits was also easy to note how the systems commuted their switches to intermittently change to the topology 1, 2 or 3.

- It was observed that the buck converter controlled by sine waveform despite its different
$T$–periodic signal (it was changed the ramp for a sine wave), can regulate as well as the buck converter controlled by ramp, but exhibits dissimilar behaviors.

- Making the first comparisons, we can conclude according to the simulation results of both systems presented above, that the buck converter controlled by sine wave becomes stable faster than the buck converter controlled by ramp, this possibly due to the system avoids the non-smooth regions that the ramp has, so keeping away from border collision bifurcations or other non-smooth bifurcations.
4 One-dimensional bifurcation diagrams

Abstract

In this chapter is computed and analyzed bifurcation diagrams for the buck converter controlled by ramp and for the buck converter controlled by sine waveform. The bifurcation diagrams are obtained from the most important parameters of each system. Phase portraits, stroboscopic maps and time responses are acquired in order to find, understand and observe the bifurcations dynamic that have not been previously described. Furthermore, some comparisons between the systems under consideration are made.
Bifurcation diagrams show the changes in the system behavior when a parameter is varied. All the diagrams that will be shown later were built with the same simulation method, by events, obtained iterating the system for each parameter value, sampling the state variable “\(v(t)\)” once per cycle and storing the last 100 points to be plotted, then, taking the last value of “\(v(t)\)” \((V(t_{end}))\) and “\(i(t)\)” \((I(t_{end}))\) as initial conditions for the next iteration with the next parameter value.

## 4.1 Buck converter controlled by sine waveform

To observe and determine the behavior of the Figure 2-5 circuit under variation of its most important parameters, in this section can be found a bifurcation analysis. Moreover, the bifurcation diagrams are acquired sweeping the parameters ascending and descending in order to make a basins of attraction study in other chapter.

All the bifurcation diagrams obtained below were built keeping constant this parameters: \(Vu = 8.2V\), \(V_{Lo} = 3.8V\), \(T = 400\mu s\), \(a = 8.4\).

### 4.1.1 “Vin” as bifurcation parameter

In the Buck converter \(Vin\) has been the main parameter of variation; therefore, we have computed two bifurcation diagrams taking \(Vin\) as bifurcation parameter, one taking \(Vin\) ascending and the other one taking \(Vin\) descending; they are shown in Figure 4-1. The component values used can be seen in Table 4-1.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>L</td>
<td>20mH</td>
<td>R</td>
<td>22Ω</td>
</tr>
<tr>
<td>C</td>
<td>47(\mu)F</td>
<td>Vref</td>
<td>12V</td>
</tr>
</tbody>
</table>

Table 4-1: Table of component values to compute the bifurcation diagrams varying \(Vin\).

Looking at Figure 4-1 and comparing both bifurcation diagrams can be concluded that there are coexistence of solutions in the range \(Vin = [49.18, 51.69]V\), topic that will be analyzed in the next chapter.

On the other hand, as can be seen from Figure 4.1(a), between \(Vin = 13V\) and \(Vin = 15.21V\) the system starts with a chaotic behavior, where it can be seen a chaotic zone with sharp peaks; after this, a non-smooth change occurs, a transition from chaos to period–1 orbit is followed due to a tangent bifurcation, more specifically, due to a 5–type grazing bifurcation.

In Figure 4.2(a) is shown the attractor of the chaotic behavior before mentioned, which presents the same sharp picks observed in the bifurcation diagram, and in Figure 4.2(b) is
4.1 Buck converter controlled by sine waveform

(a) $V_{in}$ ascending.

(b) $V_{in}$ descending.

Figure 4-1: Bifurcation diagrams of $V_C$ taking $V_{in}$ as bifurcation parameter.
observed the 5–type grazing bifurcation, where $V_{co}$ grazes the low voltage ($V_{Lo}$) of the sine waveform causing the transition from chaos to period-1 orbit.

![Figure 4-2: Chaotic attractor undergoing a tangent bifurcation.](image)

Then, the period-1 orbit becomes stable and remains stable until $Vin = 38.42V$, where the periodic orbit looses stability causing a change from period-1 orbit to period-2 orbit through a saddle-node bifurcation. Afterwards the period-2 orbit remains stable for a very few $Vin$ values, until $Vin = 38.45V$, where it looses stability bifurcating immediately into chaos, this sudden change due to a tangent bifurcation; the branch of chaos observed which expands in the system until $Vin = 51.68V$ is bounded and undergoes a small growth of voltage as $Vin$ increases up to a maximum range of $V_C = [12.66, 14.14]V$.

![Figure 4-3: Tangent bifurcation leading the system to a chaotic attractor.](image)
The chaotic attractor, and the tangent bifurcation occurred in \( Vin = 38.45V \) are depicted in Figure 4-3. Besides, in Figure 4.3(b) a 3–type grazing bifurcation is presented.

Continuing, as can be noticed from Figure 4.1(a), in \( Vin = 51.68V \) another non-smooth change from chaos to period-3 orbit takes place again due to a tangent bifurcation, it is shown in Figure 4-4.

![Figure 4-4: tangent bifurcation and the resulting period-3 orbit.](image)

(a) \( Vco \) and \( Vs \) showing a 3–type grazing bifurcation.  
(b) Period-3 orbit taking \( Vin = 55V \).

The period-3 orbit remains stable until \( Vin = 58.62V \), where looses stability and the period-3 attractor undergoes a period doubling cascade as the parameter \( Vin \) is increased, observing period-6, period-12, and even period-22 orbits till the main attractor touches the chaotic saddle; as a result the chaotic saddle becomes stable spanning over the range covered by the main attractor, in this way the chaotic attractor is formed and can be seen in Figure 4-5.

![Figure 4-5: Poincaré map obtained with 40000 iterations for \( Vin = 69V \).](image)
Concluding, we can say that looking at the 3 different attractors observed before, they get smoother as $Vin$ is increased. Furthermore, the tangent bifurcations predominate and cause many of the complex dynamics.

4.1.2 “C” as bifurcation parameter

Varying $C$ ascending and descending has been found another kind of bifurcations and the presence of coexistence of solutions, which means that in the same parameter value could exist two or more attractors, and depending on the initial conditions the system can be attracted for one or other. Figure 4-6 shows two bifurcation diagrams, one of them obtained taking $C$ ascending, and the other taking $C$ descending, both were built with the component values organized in Table 4-2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$L$</td>
<td>20mH</td>
<td>$R$</td>
<td>100Ω</td>
</tr>
<tr>
<td>$Vin$</td>
<td>20V</td>
<td>$V_{ref}$</td>
<td>12V</td>
</tr>
</tbody>
</table>

Table 4-2: Table of component values to compute the bifurcation diagrams varying $C$.

As can be seen from Figure 4.6(a), $C$ has been varied from $4 \mu F$ to $100 \mu F$ in an ascending way, where from $C = 4 \mu F$ to $9 \mu F$ the system presents a period–1 orbit, posteriorly it undergoes a period doubling bifurcation, emerging a period–2 orbit which remains stable until $C = 12.23 \mu F$; then, a period doubling cascade appears, but before it becomes chaotic, because of a tangent bifurcation, the system suddenly changes the behavior into a period–2 orbit, this orbit remains stable until reaches $C = 22.26 \mu F$, where again a period doubling bifurcation occurs and the main orbit becomes period–4. At $C = 22.54 \mu F$ another non-smooth bifurcation takes place, a tangent bifurcation, making the period–4 orbit turn into a period–1 which remains stable for the rest of $C$ values.

In Figure 4.6(b), $C$ has been varied from $100 \mu F$ to $4 \mu F$, namely it has been varied in a descending way. In that diagram is observed a period–4 orbit from $C = 100 \mu F$ to $87.11 \mu F$, but at $C = 91.2560 \mu F$, the orbit undergoes a sudden jump because $V_{co}$ grazes $V_{s}$; although a tangent bifurcation occurs, the period–4 orbit tends to the same attractor; as a result, the system solution continues evolving as period–4 orbit. Then, the period–4 orbit becomes period–1, followed by period–2 and finally by a period–1 orbit, all of them emerge from tangent bifurcations as $V_{co}$ grazes $V_{s}$.

Both graphs in Figure 4-6 represent the same bifurcation diagram, but obtained varying $C$ in two different ways, this with the aim of knowing if the system tends to the same attractor
4.1 Buck converter controlled by sine waveform

Figure 4-6: Bifurcation diagrams of $V_C''$ taking $C$ as bifurcation parameter.
taking different initial conditions; thus, the diagrams previously acquitted show coexistence of solutions due to the presence of different attractors in the same $C$ value, such phenomenon requires an analysis of domains of attraction, but this analysis will be attended in the next chapter.

### 4.1.3 “L” as bifurcation parameter

The inductance in the system is directly involved with the current, and when the current drops to zero non-smooth bifurcations could appear, current–type bifurcations that will be discussed later. In Figure 4-7 can be seen a variety of bifurcation phenomena obtained with the component values shown in Table 4-3

<table>
<thead>
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<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$C$</td>
<td>$47\mu F$</td>
<td>$R$</td>
<td>$100\Omega$</td>
</tr>
<tr>
<td>$V_{\text{in}}$</td>
<td>$20V$</td>
<td>$V_{\text{ref}}$</td>
<td>$12V$</td>
</tr>
</tbody>
</table>

**Table 4-3:** Table of component values to compute the bifurcation diagrams varying $L$.

In Figure 4-7 $L$ was varied from $0.5mH$ to $12mH$ in both diagrams, but sampling the capacitor voltage for the bifurcation diagram of Figure 4.7(a) and the inductor current for the bifurcation diagram of Figure 4.7(b). Now, if we look at the range $L=[0.5, 1.45]mH$ in Figure 4.7(a), can be seen a diversity of complex behaviors (which will be analyzed later); and if we compare the same range in Figure 4.7(b), it is observed that the inductor current is always zero as $L$ increases, this because the system works in $DCM$ for almost all the switching cycle; therefore, giving a point in zero when the current is sampled every $T$.

Looking at Figure 4.7(b), can be noted that there is a period–1 orbit evolving from $L = 0.5mH$ to $7.5mH$ as the inductor current also rises; this due to the fact that the current falls to zero for an increasingly shorter time. Immediately, if it is observed Figure 4.7(a), there is also a period–1 orbit in the range $L=[1.45, 7.5]mH$. And if we look at both diagrams, the period–1 orbit turns into a period–2 as a result of a non–smooth bifurcation since $I_L$ grazes zero (see Figure 4-8 for a sketch).

The period–2 orbit turns again into period–1 orbit at $L = 9.488mH$ due to another current–type bifurcation (see Figure 4-9). The period–1 orbit remains stable until reaches the last $L$ value.
4.1 Buck converter controlled by sine waveform

Figure 4-7: Bifurcation diagrams taking $L$ ascending.

(a) $L$ as bifurcation parameter. The sampled voltage $V_C$ is plotted vertically.

(b) $L$ as bifurcation parameter. The sampled current $I_L$ is plotted vertically.
Going back to the zone that we have not analyzed yet, between $L = 0.5mH$ and $1.45mH$ in Figure 4.7(a), it is necessary to say that in that zone (as can be seen in Figure 4.7(b)) the current stays in zero (DCM) for almost all the switching cycle, it means that the current is not permanently zero in the time domain, the current presents peaks that can not be seen in the bifurcation diagram. For a closer scrutiny we present a closeup of the aforementioned zone in Figure 4-10, where $L$ was varied ascending and descending.

Looking at both diagrams (see Figure 4-10), it can be seen that there exist coexistence of solutions in the range $L=[1.3985, 1.45]mH$. Moreover, from both diagrams can be deduced
4.1 Buck converter controlled by sine waveform

Figure 4-10: Bifurcation diagrams taking $L$ as bifurcation parameter.

(a) Bifurcation diagram taking $L$ ascending.

(b) Bifurcation diagram taking $L$ descending.
that exist similar behaviors between $L = 0.6mH$ and $1.33mH$, where occurs some jumps of the main attractor due to tangent bifurcations. Within this range the signal $V_{co}$ tries to follow $V_s$ behaving in a similar way, for this reason, $V_{co}$ sometimes grazes $V_s$ causing a tangent bifurcation; as a consequence, some sudden jumps or changes in periodicity appear (Figure 4-11 outlines the $V_{co}$ and $V_s$ behavior).

![Figure 4-11: $V_{co}$ grazing $V_s$. 3-type grazing bifurcation.](image)

For a better examination of the complex behavior shown in Figure 4-10, between $L = 1.33mH$ and $1.45mH$, a closeup is presented in Figure 4-12. Both bifurcation diagrams present the same structure in the range $L=[1.33, 1.3985]mH$, i.e., there are not coexistence of solutions within those values.

As can be seen from Figure 4-10, the period–2 orbit at $L = 1.33mH$ turns due to a tangent bifurcation into a period–6 periodic orbit, which is best seen in Figure 4-12; then, the period–6 orbit undergoes a period doubling cascade until reaches $L = 1.3410mH$, but before let the period doubling cascade evolves to chaos, another tangent bifurcation takes place and a period–4 orbit emerges, remaining stable until $L = 1.3495mH$, where a cascade of tangent bifurcations turns the main attractor into a chaotic orbit. The chaotic behavior produced by a cascade of tangent bifurcations can not evolve and finishes at $L = 1.3577mH$ due to another tangent bifurcation; thus, a period–2 orbit appears and keeps stable until reaches $L = 1.3639mH$, then the period–2 orbit is followed by a period doubling cascade which is beyond interrupted by tangent bifurcations; finally, this behavior continues in an alternating way, period doubling cascades sometimes interrupted by tangent bifurcations; therefore, observing periodic and chaotic regions.
4.1 Buck converter controlled by sine waveform

(a) Bifurcation diagram taking $L$ ascending.

(b) Bifurcation diagram taking $L$ descending.

Figure 4-12: Bifurcation diagrams taking $L$ as bifurcation parameter.
In Figure 4.12(a) (varying $L$ upward), at $L = 1.45mH$ the chaotic behavior is followed by a period–1 orbit due to a tangent bifurcation, but in Figure 4.12(b) (varying $L$ downward), the period–1 orbit remains stable for more values of $L$, so turning into chaos, due to a tangent bifurcation at $L = 1.3985mH$; afterwards, the system continues evolving as the bifurcation diagram from Figure 4.12(a).

According to the above, in the range $L = [1.3985, 1.45]mH$ there exist coexistence of solutions because in Figure 4.12(a), the system presents bands of chaos and periodic orbits as $L$ increases, whereas in Figure 4.12(b), the system only presents a period–1 periodic orbit as $L$ decreases. The domains of attraction in that range will be studied in the next chapter.

### 4.1.4 “$R$” as bifurcation parameter

The system load is very significant because the Buck converter has to behave in a stable way independently of load variations. According to this, we computed a bifurcation diagram taking $R$ as bifurcation parameter. At the beginning, as $R$ was varied, the system did not present any kind of bifurcations because this was greatly robust; but changing some parameter values, it was found the bifurcation diagram shown in Fig. 4-13; the component values used are shown in Table 4-4.

<table>
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<tr>
<td>$Vin$</td>
<td>13V</td>
<td>$Vref$</td>
<td>12V</td>
</tr>
</tbody>
</table>

**Table 4-4:** Table of component values to compute the bifurcation diagram varying $R$.

It can be noted from Fig. 4-13 that there are not changes in both bifurcation diagrams, concluding that the system does not undergo coexistence of solutions to load variations.

Looking at the range $R = [2, 7.52]Ω$ in Fig. 4.13(a), it can be observed a period–1 periodic orbit, but in the range $R = [5.32, 5.44]Ω$, the system has two stable behaviors coexisting, the first attractor is period–1 and the second is period–3, hence, depending on the initial condition the system is attracted to the period–1 or period–3 orbit.

Then, at $R = 7.52Ω$, because of a tangent bifurcation the period–1 orbit turns into period–2, which remains stable until reaches $R = 67Ω$. Going back to the range $R = [7.76, 20.92]Ω$, it can be observed coexisting attractors appearing and disappearing as $R$ increases. Now, at
4.1 Buck converter controlled by sine waveform

(a) $R$ ascending. $R$ was varied from 2Ω to 200Ω.

(b) $R$ descending. $R$ was varied from 200Ω to 2Ω.

Figure 4-13: Bifurcation diagrams taking $R$ as bifurcation parameter.
$R = 67\Omega$ a period doubling bifurcation turns the period–2 orbit into period–4 just before the chaos, which quickly undergoes cascade of tangent bifurcations, transforming the system behavior into chaos until $R$ reaches the last value; during the chaos evolution, sometimes periodic bands appear caused by tangent bifurcations.

In Fig. 4-14 is illustrated the chaotic attractor for $R=198.24\Omega$, which has also been computed to show its structure.

![Graph](image)

**Figure 4-14:** Poincaré map for $R = 198.24\Omega$, obtained with 50000 iterations.

### 4.1.5 “Vref” as bifurcation parameter

Analyzing the system behavior when $V_{ref}$ is varied provides security in choosing an appropriate value for the reference so as to avoid the chaotic zones. Figure 4-15 shows the bifurcation diagrams for $V_{ref}$, the component values used are shown in Table 4-5.

<table>
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<tr>
<th>Parameter</th>
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<th>Value</th>
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<tbody>
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<td>47$\mu$F</td>
</tr>
<tr>
<td>$V_{in}$</td>
<td>20V</td>
<td>$R$</td>
<td>22$\Omega$</td>
</tr>
</tbody>
</table>

**Table 4-5:** Table of component values to compute the bifurcation diagram varying $V_{ref}$.

First of all, from Figure 4-15 can be seen qualitatively the same characteristics in both bifurcation diagrams, hence it can be noticed that there are not coexistence of solutions to $V_{ref}$ variations.
4.1 Buck converter controlled by sine waveform

Figure 4-15: Bifurcation diagrams taking $V_{ref}$ as bifurcation parameter.

(a) $V_{ref}$ ascending. $V_{ref}$ was varied from 2V to 19V.

(b) $V_{ref}$ descending. $V_{ref}$ was varied from 19V to 2V
Furthermore, in these diagrams can be observed two regions of chaos in the ranges $V_{ref}=[2, 3.45]V$ and $V_{ref}=[15.15, 19]V$, where due to a tangent bifurcations, the region of chaos turns into a period–1 orbit and the period–1 orbit turns into chaos respectively. Consequently, from the bifurcation diagrams can be concluded that is more favorable to take values within the range ($V_{ref}=[3.45, 15.15]V$) where there are not chaotic regimes, this in order to make the system operate adequately.

The chaotic attractors shown in Figure 4-16 come from the respective chaotic areas seen in the bifurcation diagrams afore mentioned.

![Chaotic attractors observed in Figure 4-15.](image)

Figure 4-16: Chaotic attractors observed in Figure 4-15.

4.2 Buck converter controlled by ramp

It has been analyzed the Buck converter controlled by ramp (see Figure 2-4) under variation of its most important parameters as in the previous section. Moreover, the bifurcation diagrams are acquired sweeping the parameters ascending and descending in order to observe the presence of coexisting attractors.

All the bifurcation diagrams obtained below were built keeping constant this parameters: $V_u=8.2V$, $V_{Lo}=3.8V$, $T=400\mu s$, $a=8.4$.

4.2.1 “Vin” as bifurcation parameter

Since many studies about bifurcations in this system through $Vin$ alterations have been reported in the past years (see [12, 16, 21, 7, 8, 30, 6, 4]), we have computed bifurcation
4.2 Buck converter controlled by ramp

diagrams varying $Vin$ ascending and descending in a broader range in order to observe new phenomenons and to do some comparisons between the two system under study in this thesis, for such reason the following bifurcation diagrams were obtained with $Vref = 12V$ as we did with the buck converter control by sine wave in the past section.

Table 4-6 shows the component values used and in Fig. 4-17 are shown the resulting bifurcation diagrams obtained taking $Vin$ as bifurcation parameter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>20mH</td>
<td>$C$</td>
<td>47µF</td>
</tr>
<tr>
<td>$Vref$</td>
<td>12V</td>
<td>$R$</td>
<td>22Ω</td>
</tr>
</tbody>
</table>

Table 4-6: Table of component values to compute the bifurcation diagram varying $Vin$.

As can be seen from Figure 4-17, both bifurcation diagrams seem to have identical characteristics, but observing precisely it can be noted that some attractors appear in one, and disappear in the other. For instance, looking at $Vin = 57V$ in Figure 4.17(b) a periodic band appears, but looking at Figure 4.17(a) the periodic band disappears. Moreover, at $Vin = 68.6V$ in Figure 4.17(a) can be seen other periodic band which can not be seen in Figure 4.17(b); therefore, we conclude that the system presents coexistence of solutions. According to this, it can be noted how some attractors appear or how others tend to disappear in the system depending on the initial conditions. Since the chaotic behavior is very sensitive to initial conditions, the system can evolve according to the main attractor, but can be also attracted by other coexisting solutions when its parameters or its initial conditions experience little changes.

As reported in the references afore mentioned, in Figure 4-17 can be seen period–1 orbits, –2, –4, –8 and –16; then the system turns into chaos to continue evolving basically with the same chaotic attractor until the last $Vin$ value. In Figure 4-18 can also be found the Poincaré map for $Vin = 35V$, and as can be observed, the chaotic attractor is the same than the attractor reported in the afore mentioned references.

4.2.2 “C” as bifurcation parameter

In the past, it has not been reported bifurcations varying $C$ in this system, this because the capacitance does not present significant variations; despite of this issue, we compute bifurcation diagrams due to the fact that many complex behaviors can be found, behaviors that let us understand the dynamic of the systems. In addition, from bifurcation diagrams it is possible to choose appropriate parameter values for desired responses, either chaotic or periodic responses.
(a) $V_{in}$ ascending. $V_{in}$ was varied from 13V to 70V.

(b) $V_{in}$ descending. $V_{in}$ was varied from 70V to 13V.

Figure 4-17: Bifurcation diagrams taking $V_{in}$ as bifurcation parameter.
4.2 Buck converter controlled by ramp

In Table 4-7 is found the parameter values selected to obtain the bifurcation diagrams depicted in Figure 4-19.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>20mH</td>
<td>$V_{in}$</td>
<td>20V</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>12V</td>
<td>$R$</td>
<td>100Ω</td>
</tr>
</tbody>
</table>

**Table 4-7**: Table of component values to compute the bifurcation diagram varying $C$.

As can be seen from Figure 4-19, both bifurcation diagrams have the same characteristics, note that only around $C = 13.8\mu F$ can be observed a little difference, where the chaotic behavior tend to a periodic band in Figure 4.19(b), however as border collisions occur frequently, the system continues evolving in a chaotic behavior. On the other hand, around $C = 30.2\mu F$ is observed how the period doubling cascade is obstructed by a border collision bifurcation, making the system suddenly turns from chaotic behavior to a period–8 orbit. Accordingly, most of the sudden changes in the system response are caused by border collision bifurcations, a nonlinear phenomena provoked by collisions of $Vco$ with the non-smooth upper and low voltage ($V_u$ and $V_{lo}$ respectively) in $V_{ramp}$. Particularly in this case, $Vco$ undergoes more oscillations as $C$ decreased, as a result, the system response has high probabilities of colliding with the non-smooth zones of $V_{ramp}$; therefore, for low $C$ values the system response tend to be chaotic.
One-dimensional bifurcation diagrams

(a) $C$ ascending. $C$ was varied from 4µF to 100µF.

(b) $C$ descending. $C$ was varied from 100µF to 4µF.

Figure 4-19: Bifurcation diagrams taking $C$ as bifurcation parameter.
In Figure 4-20 can be observed an example of border collision bifurcation (C–type) for $C = 4.3 \mu F$. Note that in this value the orbit is losing stability, for this reason the transients are longer. And when the system reaches $C = 4.4 \mu F$ suddenly the chaotic behavior becomes stable.

![Figure 4-20: $V_{co}$ and $V_{ramp}$ undergoing a C–type border collision bifurcation in $C = 4.3 \mu F$. 400T are plotted.](image)

### 4.2.3 “L” as bifurcation parameter

As it was mentioned in the past subsection for the capacitor, the inductance in the systems are not usually analyzed since this parameter does not fluctuate in significant ranges, but in this case, in order to observe and describe the complex behavior exhibited by the system controlled by ramp, $L$ was varied in the range $[0.5, 12] mH$ ascending and descending. The component values used can be seen in Table 4-8, and the respective bifurcation diagrams can be found in Figure 4-21.
(a) $L$ ascending. $L$ was varied from 0.5mH to 12mH.

(b) $L$ descending. $L$ was varied from 12mH to 0.5mH.

Figure 4-21: Bifurcation diagrams taking $L$ as bifurcation parameter.
### 4.2 Buck converter controlled by ramp

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>47µF</td>
<td>Vin</td>
<td>20V</td>
</tr>
<tr>
<td>Vref</td>
<td>12V</td>
<td>R</td>
<td>100Ω</td>
</tr>
</tbody>
</table>

**Table 4-8:** Table of component values to compute the bifurcation diagram varying $L$.

From Figure 4-21, it can be seen that both bifurcation diagrams are alike, it suggests that the system does not have coexistence of solutions and is not sensitive to initial conditions under inductance variations.

Then, it can be observed that most of the bifurcations are non-smooth as sudden leaps and inflections come about. Note that from $L = 0.5mH$ to around $6.5mH$ the system response shows similar complex behaviors, only the orbit amplitude expands as $L$ increases, or reduces as $L$ decreases. What happens is that within those $L$ values the system response presents great oscillations, causing multiple pulsing and hence higher probabilities of border collision bifurcations. For a precisely and accurate observation we will analyze the behavior response from $L = 2.3mH$ to $L = 6.5mH$, where a period-2 orbit is presented in both cases. This because all the phenomenons observed before $L = 2.3mH$ are produced for the same reasons as the phenomenons observed within the range aforementioned.

As can be observed, the period-2 periodic orbit in $L = 2.3mH$ afterwards becomes a period-1 periodic orbit due to an ordinary period-doubling bifurcation (smooth bifurcations). Then, the period-1 periodic orbit continue evolving until suddenly turns into a chaotic behavior in $L = 3.43mH$. As we defined in Section 2.5.1, this kind of bifurcation comes out through a C-type border collision bifurcation where the switching sequence is disrupted and the system response crosses the border and becomes unstable (As it was explained in Figure 2-9). Such a border collision can bee seen in Figure 4-22.

![Figure 4-22: $V_{co}$ and $V_{ramp}$ undergoing a C-type border collision bifurcation in $L = 3.43mH$.](image)
Once the system response becomes chaotic, the attractor evolves as a classic chaotic attractor produced by a C-type border collision bifurcation; soon after, the chaotic attractor passes into a period-4 chaotic bands, and in $L = 4.93mH$ due to a D-type border collision bifurcation the period-4 chaotic bands are rapidly replaced by a period-2 periodic orbit. Figure 4-23 shows how one of the four chaotic points crosses the border (D-type) for then coming to be a stable period-2 periodic orbit.

![Figure 4-23](image)

**Figure 4-23:** $V_{co}$ and $V_{ramp}$ undergoing a D–type border collision bifurcation in $L = 4.93mH$.

Not long enough, the period-2 periodic orbit experiences another non–smooth bifurcation in $L = 6.485mH$, where it can be seen an inflection in the bifurcation diagram, this due to a D–type border collision bifurcation. It means, as we have explained in Section 2.5.1, that the system crossed the border and remained stable presenting a period–2 periodic orbit. In other words, with the D–type border collision bifurcation taking place, the relative durations of the on and off intervals in $u(t)$ are disturbed, but the same switching sequence is maintained. In Figure 4-24 can be observed the D–type border collision bifurcation occurred in $L = 6.485mH$.

![Figure 4-24](image)

**Figure 4-24:** $V_{co}$ and $V_{ramp}$ undergoing a D–type border collision bifurcation in $L = 6.485mH$. 

Then, we can observe how the period–2 periodic orbit bifurcates into a period–4 periodic orbit in \( L = 7.109 \text{mH} \) resulting from a non–smooth bifurcation. But in this case because one of the two orbits grazes the zero, it means that the inductor current grazes the value zero. It can be seen in Figure 4-25, where it can be observed the period–2 orbit, afterwards the period–2 orbit undergoing the non-smooth bifurcation, and finally the resulting period–4 periodic orbit.

![Figure 4-25](image.png)

(a) Before \( L = 7.109 \text{mH} \).
(b) In \( L = 7.109 \text{mH} \).
(c) After \( L = 7.109 \text{mH} \).

**Figure 4-25**: Non–smooth bifurcation when \( i(t) \) grazes the zero in \( L = 7.109 \text{mH} \).

Going on in the bifurcation diagram, again in \( L = 7.529 \text{mH} \) appears an inflection. As we explained before, this inflection comes out due to a D–type border collision bifurcation, but the system continues stable in the period–4 orbit. The D–type border collision bifurcation can be seen in Figure 4-26.

After the period–4 periodic orbit, the system is followed by bands of chaos and windows of periodicity. To explain this phenomena, we want to highlight the two different bands of chaos and the widest periodic window. The first band of chaos begins in \( L = 7.952 \text{mH} \), where the
Figure 4-26: $V_{co}$ and $V_{ramp}$ undergoing a D–type border collision bifurcation in $L = 7.529mH$.

period–4 orbit bifurcates into chaos after a non–smooth bifurcation produced because $i_L(t)$ grazes the zero (this can be seen in Figure 4-27).

Figure 4-27: Non–smooth bifurcation when $i(t)$ grazes the zero in $L = 7.952mH$. 
The second band of chaos begins in $L = 10.04\, mH$ after a classic period doubling cascade. And between them there is the widest periodic window, starting in $L = 8.846\, mH$ (where chaos turns into a period–2 orbit after a D–type border collision bifurcation that can be seen in Figure 4-28) and ending in $L = 9.875\, mH$ with the period doubling cascade.

Within this periodic window, a particular phenomena appears, two period doubling bifurcations take place close to each other. One of them in $L = 9.1970\, mH$ when $L$ is ascending and the other one in $L = 9.5\, mH$ when $L$ is descending. Through these bifurcations the period–2 orbit becomes a period–4 orbit. Then, one of the two attractors should become the main and go on evolving as a period doubling cascade, or should collide, loose stability and become a different attractor; however, the attractors collide and remain stable in the period–4 orbit due to a C–type border collision bifurcation which can be seen in Figure 4-29.

Finally, referring to the two bands of chaos aforementioned, it can be observed that both exhibit set of windows of periodicity. In [43] the authors believe that the bands of chaos produced by period doubling cascade (smooth bifurcations) always exhibit a dense set of periodic windows. On the contrary, they say that the bands of chaos produced by border collision bifurcations (non–smooth bifurcations) are apparently “solid”, it means that there are an absence of periodic windows (as can be seen in the band of chaos located close to $L = 3.43\, mH$); therefore, they believe that this is a common feature in a wide class of non–smooth systems.

On the other hand, for our case and as we said before, both bands of chaos exhibit periodic windows although one of them is produced by non–smooth bifurcations and the other one is produced by smooth bifurcations. This allows us to conclude that for this particular system,
Figure 4-29: $V_{co}$ and $V_{ramp}$ undergoing a C–type border collision bifurcation in $L = 9.3350mH$.

the buck converter controlled by ramp, periodic windows can appear during bands of chaos produced by smooth or non–smooth bifurcations. Periodic windows that come out due to tangent bifurcations, either any type of border collision bifurcations or the class of non-smooth bifurcations brought out when the inductor current ($i(t)$) grazes zero. Particularly for this system, we have observed that most of the periodic windows result from C–type or D–type border collision bifurcations.

4.2.4 “R” as bifurcation parameter

The load is a common bifurcation parameter since this represents significant importance when designing applications. Accordingly, it has been computed bifurcation diagrams taking $R$ ascending and descending in the range $R = [2, 200]Ω$ using the component values of Table 4-9. And the bifurcation diagrams can be seen in Figure 4-30.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$2μF$</td>
<td>$V_{in}$</td>
<td>13V</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>12V</td>
<td>$L$</td>
<td>50mH</td>
</tr>
</tbody>
</table>

Table 4-9: Table of component values to compute the bifurcation diagrams varying $R$. 
4.2 Buck converter controlled by ramp

(a) $R$ ascending. $R$ was varied from $2\Omega$ to $200\Omega$.

(b) $R$ descending. $R$ was varied from $200\Omega$ to $2\Omega$.

**Figure 4-30**: Bifurcation diagrams taking $R$ as bifurcation parameter.
From Figure 4-30 can be seen that both bifurcation diagrams are similar, there is just a short portion of chaos that is less dense in Figure 4.30(b) than in Figure 4.30(a), which means that coexistence of solutions appear in the range \( R = [61.12, 64.64] \Omega \). This tell us that the system is robust against initial condition variations for most of the \( R \) values.

As in the rest of the features in the bifurcation diagrams are the same, we will analyze the bifurcation behavior in general for both diagrams. Firstly, it can be seen that an apparently non-smooth bifurcation gives rise to a period–2 orbit from a period–1 orbit close to \( R = 23.16 \Omega \) because it can be seen a leap from the period–1 orbit to the period–2 orbit, yet it is just a classic period doubling bifurcation that occurs as fast that seems to be a sudden jump. Then, the period–2 orbit is followed by a band of chaos produced by an apparently non-smooth bifurcation since the period–2 orbit seems to jump into chaos. But in this case the period–2 orbit bifurcates into chaos after pass through period–4 orbits, period–8 and so on, as a result of a classic period doubling cascade. This behavior takes place very fast, that is why the bifurcation looks like a non–smooth one. In addition, the chaotic range looks very solid, without any periodic windows. This is because once the period doubling cascade reaches the chaotic regime, C–type and D–type border collision bifurcations occur letting the system be stable in the chaotic orbit.

Nevertheless, through a C–type border collision bifurcation close to \( R = 64.64 \Omega \) the chaotic regime can not continue stable and bifurcates into a period–3 orbit (see Figure 4-31).

![Figure 4-31: Vco and Vramp undergoing a C–type border collision bifurcation in R = 64.64Ω.](image)

Seemingly the period–3 orbit continues stable until reaches the end of \( R \); however, close to \( R = 152.68 \Omega \) a sudden leap or a discontinues change appears. A D–type border collision bifurcation which can be seen in Figure 4-32 makes the system bifurcate into the same period–3 orbit, it means that the system crosses the border and remains stable undergoing a period–3 behavior until the last \( R \) value.
4.2 Buck converter controlled by ramp

4.2.5 “Vref” as bifurcation parameter

Varying the reference voltage (Vref) let us know the range within which the system can reduce the input voltage without exhibit complex behaviors; additionally, we want to find and describe the overall bifurcation phenomena in the system. Conforming to this, two bifurcation diagrams have been computed taking the parameter value ascending and descending for the same reason explained in the past sections, in order to find coexistence of solutions. Vref was varied from 2V to 19V using the component values of Table 4-10 and the corresponding bifurcation diagrams can be found in Figure 4-33.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>47µF</td>
<td>Vin</td>
<td>20V</td>
</tr>
<tr>
<td>R</td>
<td>22Ω</td>
<td>L</td>
<td>20mH</td>
</tr>
</tbody>
</table>

Table 4-10: Table of component values to compute the bifurcation diagrams varying Vref.

Initially, From Figure 4-33 can be noted that both bifurcation diagrams are completely similar, which means that there are not coexistence of solutions in the system when Vref is varied. Secondly, there is two regions of chaos, one at the beginning of the diagram and the other one at the end of the diagram. Thirdly, both regions of chaos are followed of period doubling cascades which then end up in a period–1 orbit. It should be emphasized that both regions of chaos exhibit periodic windows due to tangent bifurcations, which in this case are mainly D–type or C–type border collision bifurcations that create stable periodic orbits. And finally, we can conclude that the system can regulate appropriate within the range Vref = [4.635, 14.445]V since there are not complex behaviors.
(a) $V_{ref}$ ascending. $V_{ref}$ was varied from 2V to 19V.

(b) $V_{ref}$ descending. $V_{ref}$ was varied from 19V to 2V.

Figure 4-33: Bifurcation diagrams taking $V_{ref}$ as bifurcation parameter.
4.3 Conclusions

The chaotic attractors were also computed and can be seen in Figure 4-34. Here it can be observed that the chaotic attractor in Figure 4.34(a) is smoother than the chaotic attractor of Figure 4.34(b). In addition, it can be noted that the latter attractor is very similar to the chaotic attractor of Figure 4.16(b) (obtained from the sine–controlled buck converter). This may be because in the bifurcation diagram of $V_{ref}$ computed from the ramp–controlled buck converter (see Figure 4-33), close to $V_{ref} = 18V$ and until $V_{ref} = 19V$ the structure is very similar to the structure of the bifurcation diagram of $V_{ref}$ computed from the sine–controlled buck converter (see Figure 4-15). Which means that both systems bifurcated into that attractor through a similar type of bifurcation.

![Figure 4-34: Chaotic attractors observed in Figure 4-33.](image)

(a) Poincaré map for $V_{ref} = 2V$, obtained with 40000 iterations.  
(b) Poincaré map for $V_{ref} = 19V$, obtained with 40000 iterations.

4.3 Conclusions

- It was computed bifurcation diagrams from both systems, the buck converter controlled by ramp and by sine waveform, taking as bifurcation parameters $Vin$, $C$, $L$, $R$ and $Vref$, which are the most important parameters in the systems before mentioned.

- Each bifurcation diagram was analyzed numerically describing the type of bifurcation and the resulting change in the system.

- Phase portraits and time responses were obtained in order to verify each type of bifurcation that takes place in the systems.

- Chaotic attractors were obtained to observe the dynamic of the chaotic orbits.
• The buck converter controlled by ramp mainly exhibits D–type and C–type border collision bifurcations. Grazing–type and period doubling bifurcations appear in the system as well.

• The chaotic bands produced by border collision bifurcations tend to be solid, without periodic windows, while the chaotic bands produced by smooth bifurcations tend to exhibit dense set of periodic windows. However, in the buck converter controlled by ramp, due to the non–smoothness of the ramp signal, abundant border collision bifurcations make the system exhibit periodic windows in the bands of chaos produced by smooth bifurcations.

• The buck converter controlled by sine waveform exhibits mainly smooth bifurcations as period doublings, but also grazing–type bifurcation appear in the system. This due to the smooth nature of the sine waveform. According to this, border collision bifurcations are avoided in this type of control.

• If we make a general comparison between the bifurcation diagrams of the buck converter controlled by ramp and the bifurcation diagrams of the buck converter controlled by sine waveform, it can be noted several differences; for instance, the first converter exhibits many non-smooth bifurcations; therefore, the system tends to undergo more complex behaviors or chaotic regimes, the range of periodic orbits is shorter and the probability of the emergence of bifurcations is higher. On the contrary, the second converter exhibits less non–smooth bifurcations; therefore, the system tends to undergo more periodic behaviors than complex behaviors and the probability of the emergence of bifurcations is less, thus the range of periodic orbits is longer. Nevertheless, usually in one bifurcation diagram appears more than one and different chaotic attractors.

• Coexistence of solutions were found in both systems and will be studied in other chapter. With this, we can conclude that both systems are sensitive to initial conditions. Although in the variation of some parameters it was not found solutions coexisting, in most of them it could be found at least in a short range of the parameter.

• Finally, it can be concluded that changing the $T$–periodic signal in the control law, the system dynamic changes radically, providing different behaviors and showing a completely distinct complex responses. In this case, using a $T$–periodic sine wave it was found that the system exhibits more and longer periodic windows. We believe that due to the smoothness of the sine waveform, there is a longer region that the control signal ($V_{co}$) can touch without presenting border collision bifurcations. However, when one parameter varies, tangent bifurcations appear. This suggests that no matter which type of periodic signal is used, non–smooth bifurcations of border collision and tangent type always come out.
5 Basins of Attraction Analysis

Abstract

In this chapter bifurcation diagrams that have been obtained in Section 4.1 for the buck converter controlled by sine waveform varying the parameters ascending and descending are used to find coexisting attractors, which are normally an undesired behavior in nonlinear systems. Although it is demonstrated that these bifurcation diagrams are not the sufficient remedy to find coexistence of solutions, it is also shown that they are a good tool. Once bifurcation diagrams show the range where coexistence of solutions appear, we proceed to study in that range the shape of the basins and the system solutions. All with the aim of determining the specific regions in which the system presents different behaviors. This is because many practical applications today need not only periodic solutions but also more complex ones. Finally, basins of attraction are obtained and studied for some parameters of the system. Bounded and fractal regions are observed; moreover, the evolution of the attractors in the time domain for each parameter value are shown.
Taking into account the buck converter controlled by sine waveform (see Figure 2-5) and its respective one-dimensional bifurcation diagrams obtained in Section 4.1, coexisting attractors will be search and studied. The process can be carried out while the bifurcation parameter is increased or decreased, and in both cases the resulting bifurcation diagrams must be equal; otherwise, it can be concluded that the system has more than one solution for certain values of the parameters.

### 5.1 When $V_{in}$ is the bifurcation parameter

To study domains of attraction, in this case $V_{in}$ has been taken as bifurcation parameter and its respective diagrams are shown in Figure 4-1.

In Fig. 4.1(a), $V_{in}$ was varied in an ascending way from 13V to 70V, and in Fig. 4.1(b), $V_{in}$ was varied in a descending way from 70V to 13V, but both diagrams do not show big differences between them, there is just a little disagreement in the range $V_{in} = [49.18, 51.68]$V where there exist coexistence of solutions; therefore, if we take a $V_{in}$ value, for instance $V_{in} = 50V$, it can be noted that one solution is a period-3 orbit and the other one is a chaotic orbit.

In Fig. 5-1 is shown the basins of attraction diagram for $V_{in} = 50V$, where in the bifurcation diagram with $V_{in}$ ascending the system presents a chaotic orbit, and in the bifurcation diagram with $V_{in}$ descending the system presents a period-3 periodic orbit. The diagram shows that blue and red correspond to the 3T basin and chaotic basin respectively; hence, the system undergoes a period–3 behavior in the blue zone and a chaotic behavior in the red zone. It suggests, if the system takes a initial condition located in the blue zone, the system solution tends to the three yellow points whose values are shown in Table 5-1.

<table>
<thead>
<tr>
<th>$V^*$ (volts)</th>
<th>$I^*$ (amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.7991</td>
<td>0.7569</td>
</tr>
<tr>
<td>12.1241</td>
<td>0.5539</td>
</tr>
<tr>
<td>13.6665</td>
<td>0.4784</td>
</tr>
</tbody>
</table>

**Table 5-1**: Stationary points of 3T attractor.

And if the system takes a initial condition located in the red zone, the system solution tends to the black points which represent the chaotic attractor.

What is more, in Figure 5-1 can be noted that the boundary between the two basins is not fractal because the color bands are bounded, it means that the color zones are well defined.
5.1 When $V_{in}$ is the bifurcation parameter

The phase portrait and the time domain of the $3T$ and the chaotic orbits have been simulated and can be seen in Figure 5-2.

After find these two attractors coexisting in $V_{in} = 50V$, meanwhile the bifurcation diagrams were analyzed, we casually found three solutions coexisting in $V_{in} = 20V$ (They do not appear in Figure. 4-1), demonstrating that obtaining bifurcation diagrams in an ascending and descending way is not a sufficient method to find coexisting attractors. Now, it will be analyzed domains of attraction for the parameter value before mentioned.

In Figure. 5-3 is shown the basins of attraction diagram for $V_{in} = 20V$, where in the bifurcation diagram with $V_{in}$ ascending the system presents a period–1 periodic orbit, and in the bifurcation diagram with $V_{in}$ descending the system also presents a period–1 periodic orbit; however, there are three coexisting solutions, the first one is period–1, the second one is period–3 and the third one is period–4.

The diagram in Figure. 5-3 shows that blue, red and yellow correspond to the $1T$ basin, $3T$ basin and $4T$ basin respectively; hence, the system undergoes a period–1 behavior in the blue zone, period–3 behavior in the red zone and period–4 behavior in the yellow zone. Therefore, as can be seen, if the system takes a initial condition located in the blue zone,
5 Basins of Attraction Analysis

(a) $V_{co}$ and $V_s$ showing a period–3 periodic orbit.

(b) $V_{co}$ and $V_s$ showing a chaotic orbit.

(c) Phase portrait of the period–3 orbit.

(d) Phase portrait of the chaotic orbit.

**Figure 5-2**: $3T$ and chaotic orbits coexisting in $Vin = 50V$

**Figure 5-3**: Basins of attraction for $Vin = 20V$. $V_C$ range is $[11.5V, 13.5V]$; $I_L$ range is $[0A, 1A]$. 
5.2 When $C$ is the bifurcation parameter

the system solution tends to the green point which has the value:

\[ X^* = [12.5837V, 0.5355A] \]

Or if the system takes a initial condition located in the red zone, the system solution tends to the three white points whose values are shown in Table 5-2.

<table>
<thead>
<tr>
<th>$V^*$ (volts)</th>
<th>$I^*$ (amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.4803</td>
<td>0.4712</td>
</tr>
<tr>
<td>12.9395</td>
<td>0.5620</td>
</tr>
<tr>
<td>12.3785</td>
<td>0.6249</td>
</tr>
</tbody>
</table>

**Table 5-2:** Stationary points of $3T$ attractor.

And if the system takes a initial condition located in the yellow zone, the system solution tends to the four black points whose values are shown in Table 5-3.

<table>
<thead>
<tr>
<th>$V^*$ (volts)</th>
<th>$I^*$ (amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.2858</td>
<td>0.3888</td>
</tr>
<tr>
<td>13.3499</td>
<td>0.5909</td>
</tr>
<tr>
<td>12.4474</td>
<td>0.7121</td>
</tr>
<tr>
<td>11.6936</td>
<td>0.5517</td>
</tr>
</tbody>
</table>

**Table 5-3:** Stationary points of $4T$ attractor.

In addition, in Figure. 5-3 can be noted that the boundary between the three basins is also not fractal because the color bands are bounded, it means that in the red zone can not appear blue or white basins, and the same in the other zones.

The $1T$, $3T$ and $4T$ periodic orbits have been simulated and can be seen in Figure. 5-4.

### 5.2 When $C$ is the bifurcation parameter

It was also found coexisting attractors in the bifurcation diagrams taking $C$ as bifurcation parameter, the diagrams are shown in Figure. 4-6.

In these diagrams can be seen coexisting attractors in the range $C = [8.89, 21.75] \mu F$, where as can be seen in Figure. 4.6(b), there are just period–2 periodic orbits, but in Figure. 4.6(a)
there are period–2, period–4, again period–2 and again period–4 orbits as $C$ is increased. But in this thesis we are going to analyze the range $C = [87.1120, 100] \mu F$, where there exist period–1 orbits in the bifurcation diagram taking $C$ ascending, and period–4 orbits in the bifurcation diagram taking $C$ descending. We have taken $C = 95.2 \mu F$ to analyze domains of existence, and the resulting diagram is shown in Figure. 5-5.

It is observed that in the figure aforementioned blue and red correspond to the $1T$ basin and $4T$ basin respectively, it means that the system undergoes a period–1 behavior in the blue zone and a period–4 behavior in the red zone. In other words, if the system takes a initial condition located in the blue zone, the system solution tends to the yellow point whose value is:

$$X^* = [12.5986V, 0.0890A]$$

If the system takes a initial condition located in the red zone, the system solution tends to the four black points which have the values observed in Table 5-4.
5.3 When \( L \) is the bifurcation parameter

As could be seen in the bifurcation diagrams taking \( L \) as bifurcation parameter, coexistence of solutions appear since the diagrams showed different behaviors for different initial conditions.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{\( V^* \) (volts)} & \textbf{\( I^* \) (amperes)} \\
\hline
12.8094 & 0.0476 \\
12.7626 & 0.1447 \\
12.4114 & 0.1742 \\
12.5204 & 0.0225 \\
\hline
\end{tabular}
\caption{Stationary points of 4\( T \) attractor.}
\end{table}

\textbf{Figure 5-5}: Basins of attraction for \( C = 95.2\mu F \). \( V_C \) range is \([12.3V, 12.95V]\); \( I_L \) range is \([0A, 0.3A]\).

On the other hand, the boundary between the two basins in Figure. \textit{5-5} is fractal due to blue and red zones are unbounded.

The periodic orbits 1\( T \) and 4\( T \) have been computed and can be observed in Figure. \textit{5-6}.

\textbf{5.3 When \( L \) is the bifurcation parameter}

As could be seen in the bifurcation diagrams taking \( L \) as bifurcation parameter, coexistence of solutions appear since the diagrams showed different behaviors for different initial conditions.
In Figure 4.12(a), $L$ was varied in an ascending way from $1.337\,mH$ to $1.451\,mH$, and in Figure 4.12(b), $L$ was varied in a descending way from $1.451\,mH$ to $1.337\,mH$, showing different behaviors in the range $L = [1.3985, 1.4498]\,mH$; thus, we conclude that in the range aforementioned there exist coexistence of solutions. According to this an analysis of domains of attraction is necessary.

In Figure 5-7 is shown the basins of attraction diagram computed for $L = 1.4434\,mH$, where there are two solutions coexisting. In the bifurcation diagram with $L$ ascending (Figure 4.12(a)) the system presents a period–5 periodic orbit, and in the bifurcation diagram with $L$ descending (Figure 4.12(b)) the system presents a period–1 periodic orbit.

The diagram in Figure 5-7 shows that blue and red correspond to the $1T$ basin and $5T$
5.3 When $L$ is the bifurcation parameter

Figure 5-7: Basins of attraction for $L = 1.4434mH$. $V_C$ range is $[11.5V, 13V]$; $I_L$ range is $[0A, 1.8A]$.

...basin respectively; therefore, the system undergoes a period–1 behavior in the blue zone and period–5 behavior in the red zone. Namely, if the system takes a initial condition located in the blue zone, the system solution tends to the yellow point which has the value:

$$X^* = [12.5380V, 0.2212A]$$

But if the system takes a initial condition located in the red zone, the system solution tends to the five black points which have the values shown in Table 5-5.

<table>
<thead>
<tr>
<th>$V^*$ (volts)</th>
<th>$I^*$ (amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5700</td>
<td>0.3690</td>
</tr>
<tr>
<td>12.6614</td>
<td>0.4640</td>
</tr>
<tr>
<td>12.5979</td>
<td>0.4052</td>
</tr>
<tr>
<td>12.5803</td>
<td>0.3836</td>
</tr>
<tr>
<td>12.5732</td>
<td>0.3739</td>
</tr>
</tbody>
</table>

Table 5-5: Stationary points of $5T$ attractor.

Besides, in Figure 5-7 is interesting to note that the boundary between the two basins is not
fractal because the color bands are bounded, there is a clear separation or borders between red and blue.

In addition, if we choose a $L$ value in the bifurcation diagrams where coexist the period–1 orbit and the chaotic orbit, the structure of the basins of attraction is the same as the study in Figure 5-7, just that the period-5 orbit tends to chaos, it means that the chaos basin is going to have the same structure as the $5T$ basin; thus, the red zone would be the basin of attraction of the chaotic orbit.

Finally, the $1T$ and $5T$ periodic orbits have been simulated and are shown in Figure 5-8.

![Figure 5-8](image)
(a) $Vco$ and $Vs$ showing a period–1 periodic orbit. (b) $Vco$ and $Vs$ showing a period–5 periodic orbit.

Figure 5-8: $1T$ and $5T$ periodic orbits coexisting in $L = 1.4434mH$

It is worth noting that obtaining the bifurcation diagrams shown in Figure 4-12, the system was working in DCM; accordingly, the inductor current ($i(t)$) remains in zero for a period of time. The phase portraits observed in Figure 5-9 show the explained before for the system undergoing the coexisting period–1 and period–5 solutions.

![Figure 5-9](image)
(a) Period–1 periodic orbit in steady state. (b) Period–5 periodic orbit in steady state.

Figure 5-9: $i(t)$ shows how the system enters to work in DCM.
5.4 Conclusions

- This chapter reports coexisting attractors appearing simultaneously in different parameters of a new design of the DC-DC buck converter, controlled by sine wave, affirming that the system has more than one solution for a given parameter values with different initial conditions.

- Each attractor has different periodicity, and even chaotic attractors appear turning into periodic orbits for some initial conditions; therefore, a system with an apparently periodic solution does not provide enough confidence, it is necessary a domains of attraction study.

- Bifurcation diagrams obtained varying the parameters in an ascending and descending way, was a source of great help to find coexistence of solutions, but we must to say that these diagrams are not a sufficient method and it was demonstrated; for this reason, it is sometimes necessary to do trial and error examinations, i.e., simulate the system for different initial conditions and see if more than one coexisting attractors are found, without resorting to bifurcation diagrams.

- Finally, it can be concluded that in a fractal region is more difficult to choose an initial condition for the orbit desired, the basins are very close between them and appear for all over the plane if the initial conditions matrix is increased.
6 Control Algorithms for the PMW Voltage-Controlled Buck Converter

Abstract

In this chapter an analog-controller of chaos for the dc-dc buck converter controlled by PWM is proposed. This technique is based on an adaptive control, where the $T$–periodic ramp or sine signals ($V_{\text{ramp}}$ and $V_s$ respectively) are modified to behave according to the control signal ($V_{\text{co}}$) and the input voltage ($V_{\text{in}}$) changes; all in order to extend even more the $V_{\text{in}}$ range over which the $1T$–periodic orbit remains stable. This novel strategy basically redefines $V_{\text{ramp}}$ or $V_s$ in such a way it does not need predefined constant low ($V_{\text{L0}}$) or upper ($V_u$) voltages because, although these new adaptive ramp and sine ($V_{\text{ar}}$ and $V_{\text{as}}$ respectively) are periodic, their slope or amplitude and offset voltage always change; namely, $V_{\text{ar}}$ and $V_{\text{as}}$ are designed to track $V_{\text{co}}$. Additionally, this control techniques greatly reduce the percentage of regulation error ($\%e$) as well as eliminates the orbits of period grater than one and even the chaotic behavior when $V_{\text{in}}$ is varied. Finally, numerical results are obtained for the adaptive–sine control. Also, numerical and experimental results are obtained for the adaptive–ramp control; all in order to validate the performance of the systems, not to mention it is also shown high agreement between numerical and experimental results.
6.1 DC-DC Buck Converter Controlled by Ramp

In this case, the buck power converter controlled by ramp strategy, because experimental results will be obtained, is going to be considered with the equivalent resistance of the current sensor and the internal inductor resistance. For this reason, the system is newly depicted in Figure 6-1 and described by Eq. (6-1).

\[
\begin{bmatrix}
\dot{V}_C \\
\dot{I}_L
\end{bmatrix} =
\begin{bmatrix}
-1/RC & 1/C \\
-1/L & -R_{in}/L
\end{bmatrix}
\begin{bmatrix}
V_C \\
I_L
\end{bmatrix} +
\begin{bmatrix}
0 \\
V_{in}/L
\end{bmatrix} u
\]  

(6-1)

where the state variables corresponding to the capacitor voltage and inductor current are denoted by \( V_C \) and \( I_L \). The switches \( S_1 \) and \( S_2 \) are operating in a complementary way. \( R_{in} \) is the equivalent resistance of the current sensor and internal inductor resistance. The other parameters and the controlled law are described in Section 2.2.

To perform numerical analysis, we have used Simulink, and the schematic diagram is shown in Figure 6-2. Here it is shown the buck converter circuit in closed loop used to obtain all the simulation diagrams. As can be observed, \( V_C \) has been rescaled with \( R_1 \) and \( R_2 \). On the other hand, inside of the controller box, it was built two kinds of controllers (using ramp and adaptive ramp techniques) in order to obtain all the simulation results for each and afterwards to be able to compare them.

Table 6-1 contains all the parameter values used in this section.
Figure 6-2: Simulink schematic of the buck converter used to simulate the system controlled by ramp and adaptive ramp.

Table 6-1: Parameter values for the buck converter controlled by ramp.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage ($V_{in}$)</td>
<td>(20-40)V</td>
</tr>
<tr>
<td>Reference voltage ($V_{ref}$)</td>
<td>11.3V</td>
</tr>
<tr>
<td>Inductance ($L$)</td>
<td>20mH</td>
</tr>
<tr>
<td>Inductor resistance ($R_{in}$)</td>
<td>3.8Ω</td>
</tr>
<tr>
<td>Rescale resistors (R1 and R2)</td>
<td>6.8KΩ</td>
</tr>
<tr>
<td>Capacitance ($C$)</td>
<td>47µF</td>
</tr>
<tr>
<td>Load ($R$)</td>
<td>22Ω</td>
</tr>
<tr>
<td>Lower voltage ($V_u$)</td>
<td>3.5V</td>
</tr>
<tr>
<td>Upper voltage ($V_{Lo}$)</td>
<td>8.1V</td>
</tr>
<tr>
<td>Ramp period ($T$)</td>
<td>400µs</td>
</tr>
<tr>
<td>$A_1$ gain ($a$)</td>
<td>7.8</td>
</tr>
<tr>
<td>$A_2$ gain (Comp)</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
6.1 DC-DC Buck Converter Controlled by Ramp

6.1.1 Presence of Chaos in the System

In Figure 6-3 a one-dimensional bifurcation diagram as $V_{in}$ varies is shown, this bifurcation diagram was firstly reported in 1990 by Jonathan H.B. Deane and David C. Hamill in [12], then within 1990 and 1999 other related studies derived from the mentioned bifurcation diagram appeared and some of them can be found in [21, 16, 4, 7, 8, 6, 30]. The diagram observed in Figure 6-3 differs from the others found in previous references due to it was obtained using different low and upper ramp values, and different $A1$ gain ($V_{Lo} = 3.5V$, $V_u = 8.1V$ and $a = 7.8$ respectively), causing a horizontally displacement of the bifurcation points towards the right, but conserving the same features. Accordingly, $1T$–periodic orbits, $2T$–, $4T$–, $8T$– and chaos can be seen in the diagram as described in the references aforementioned. But in this case, close to $V_{in} = 28V$ a period doubling orbit is detected and after $V_{in} = 37.5V$ chaos is presented.

With the aim to eliminate the chaotic behavior and orbits with period greater than one, a control technique based on adaptive ramp is proposed.

Figure 6-3: One-dimensional bifurcation diagram of the buck converter controlled by ramp. Capacitor voltage ($V_C(t)$) versus input voltage ($V_{in}$).
6.2 DC-DC Buck Converter Controlled by Adaptive Ramp

In order to counteract the chaos presence, we plan to build a new $T$–periodic ramp signal according to the $V_{co}$ behavior and $V_{in}$ variations, as a result, obtain a $T$–periodic adaptive ramp ($Var$) which tracks $V_{co}$. Looking at this purpose, we have developed the following analysis:

As can be observed from the $V_{co}$ signal, its behavior is extremely related to $V_{in}$ variations; it means, as $V_{in}$ increases $V_{co}$ increases as well. If we look at Figure 6-3, complex dynamics begin to appear as $V_{in}$ increases, and this occurs because $V_{co}$ exceeds the thresholds $V_{Lo}$ and $V_{U}$ in the $V_{ramp}$ signal ($V_{co}$ exceeds the range $V_{Lo} < V_{co} < V_{U}$); subsequently, the probability that a bifurcation occurs is much higher since undesired behaviors such as: skipped cycles, multiple pulsing, border collisions or grazings could emerge. Keeping in mind this observation, it has been found out that the $V_{ramp}$ slope must be changed by $V_{in}$.

At the moment, we have defined the new $V_{ramp}$ slope which is $V_{in}$, however, if we plan to adapt $V_{ramp}$ to the $V_{co}$ signal, it is also necessary to remove the $V_{ramp}$ boundaries ($V_{Lo}$ and $V_{U}$) and use a variable offset voltage. According to this, we have observed that the most suitable signal to use as offset voltage in $V_{ramp}$ is $V_{co}$, so that the new $T$–periodic ramp signal which we are going to call “adaptive ramp ($Var$)” behaves similar to the $V_{co}$ signal.

After the analysis we have stated above, it can be intuitively realized that the new $T$–periodic ramp signal, with $V_{co}$ as offset voltage and $V_{in}$ as the slope, would reach too high values in view of the fact that $V_{co}$ and $V_{in}$ acquire high values too; this issue may cause an undesired response since the system would stop to regulate. In order to deal with or adjust this undesirable high values in the new $T$–periodic ramp signal, we determined to reduce them and keep them controlled using a new term in the ramp equation, dividing $V_{co}$ and $V_{in}$ by a term which has been named “$k$”. Eventually, gathering all the information described before can be formulated the $T$–periodic adaptive ramp as follows:

$$Var(t) = \frac{V_{co}}{k} + \frac{V_{in}}{k} \frac{t}{T}$$

(6-2)

where $Var$ corresponds to the signal which will replace $V_{ramp}$; $V_{co}$ is the new offset voltage and $V_{in}$ is the new slope. From Eq. (6-2) can be also perceived that any change at any time in $V_{co}$ and $V_{in}$ immediately update $Var$, this allows that $Var$ changes at the same time $V_{co}$ changes, achieving that $Var$ adapts to $V_{co}$.

Note that the term $k$ makes the adaptive ramp waveform ($Var$) changes its offset voltage and its slope as $V_{co}$ and $V_{in}$ vary respectively.

It is emphasized that in Section 6.1 or 2.2 the control scheme is based on a fixed ramp waveform, meanwhile in this section we have considered the case where the ramp waveform is variable. Accordingly, we have simply adjusted the $T$–periodic ramp signal to produce a
$T$–periodic adaptive ramp signal as can be seen in Figure 6-4.

**Figure 6-4**: DC-DC buck converter controlled by adaptive ramp.

Summarizing, $V_{co}$ and $u$ are defined by Eqs. (6-3) and (6-4) respectively.

\[
V_{co}(t) = a \left( V_C(t) - V_{ref} \right)
\]  

\[
u = \begin{cases} 
1 & \text{if } V_{co} < V_{ar} \\
0 & \text{if } V_{co} > V_{ar} 
\end{cases}
\]  

\[6-3\]

\[6-4\]

### 6.2.1 Obtaining $k$

The objective now is to find the best value for $k$ from Eq. 6-2; this value corresponds to that guarantees $1T$–periodic solution with good regulation performance. Numerical tools such as bifurcation diagrams have been used to find a suitable value for $k$. Initially, a two-dimensional bifurcation diagram (depicted in Figure 6-5) is computed. $V_{in}$ and $k$ are used as the bifurcation parameters. The aim of this diagram is to find a region where the system presents a $1T$–periodic solution. From Figure 6-5 is observed that the always desired behavior can be achieved for any $V_{in}$ with a properly choice of $k$.

The second step is to find the values for which the system exhibits the lowest percentage of regulation error ($\%e$). Figure 6-6 shows the steady state percentage error as $V_{in}$ and $k$ vary.
Figure 6-5: Two-dimensional bifurcation diagram. Bifurcation parameters are $V_{in}$ and $k$. Color codes can be seen at the top of the diagram.

Figure 6-6: Three-dimensional diagram. Bifurcation parameters are $V_{in}$ and $k$, showing the percentage of regulation error ($\%e$). Color codes can be seen at the top of the diagram.
As can be seen from Figure 6-6, the black dashed line shows the values of $k$ for each value of $V_{in}$ that allow the system to obtain the lowest $\%e$.

In Figure 6-7 the lowest percentage error and the stability limit of $1T$–periodic orbit for each combination of $V_{in}$ and $k$ are presented; in fact, the limit of the stability coincides with the lowest percentage error. Curves were smoothed by cubic interpolation. $1T$–periodic orbits taken from Figure 6-7 upper are mapped to 6-7 lower.

![Figure 6-7](image)

**Figure 6-7:** Function that provides suitable values of $k$ according to $V_{in}$.

From Figure 6-5, Figure 6-6 and Figure 6-7 can be observed that it is possible to maintain a $1T$–periodic orbit within the $V_{in}$ range [13, 70]V with a properly choice of $k$. Then, in order to prove our control technique numerically and experimentally we have chosen $V_{in}$ to vary within the range [20, 40]V, and the first numerically computed bifurcation diagram using Eq. (6-2) is depicted in Figure 6-8.

Here it is shown that chaos and orbits of period greater than one were eliminated. Now, comparing this bifurcation diagram with the obtained with a classical ramp control (Figure 6-8 and Figure 6-3 respectively), it can be clearly seen that our proposed technique removes successfully the orbits of period greater than one and even the chaotic zone; moreover, looking at Figure 6-3, in $V_{in} = 40V$ the system response reaches a maximum value of around $V_C = 12.8V$, hence, obtaining $\%e$ with respect to $V_{ref}$ results in 13.27%; whereas looking at Figure 6-8, in $V_{in} = 40V$ the system response reaches a maximum value of around
Figure 6-8: One-dimensional bifurcation diagram of the buck converter controlled by adaptive ramp. $V_C(t)$ versus input voltage $V_{in}$.

$V_C = 12V$, hence, obtaining %e with respect to $V_{ref}$ results in 6.19%; as a result, it can be concluded that %e has decreased up to 7.08%.

6.2.2 Numerical and Experimental Comparison

To implement the adaptive ramp technique, it is necessary to use Eq. (6-2). This equation is a nonlinear function in which $k$ depends on $V_{in}$; then, there are two options to implement this controller. The first one is to use a digital device such as DSP or FPGA, and the second option is to use analog technology. As we want to implement the simplest controller which also produces excellent results, it is chosen the second option. For this reason and to make the implementation easier, it is used the controller based on adaptive ramp with variable offset voltage and fixed slope. In this case we have selected the input voltage bounded such that $V_{in} \in [20, 40]$V; therefore, $V_{in}$ and $k$ in Eq. (6-2) are fixed to 40V and 5.7 respectively. Both taken properly from Figure 6-5 and Figure 6-7 looking at the $1T$-periodic orbit and the lowest %e for all $V_{in}$ values. According to this, the simplified adaptive ramp is given by Eq. (6-5).

$$Var(t) = \frac{Vco}{5.7} + \frac{40}{5.7}V_{ramp}$$ (6-5)
Numerical Results

The one–dimensional bifurcation diagram is shown in Figure 6-9. In this diagram $V_{in}$ is the bifurcation parameter and Eq. (6-5) was used as control strategy instead of Eq. (6-2). In this bifurcation diagram it is observed that a $1T$–periodic orbit is preserved for all values of $V_{in}$, and regulation properties are obtained; the desired output voltage is $V_{ref} = 11.3V$.

In order to compare numerical and experimental results two particular examples are selected. The aim is to present undesired behaviors such as periodic orbits grater than one or chaos for the ramp–controlled system and after applying the adaptive–ramp control. For instance, in Figure 6.10(a) – Figure 6.10(c) it can be seen a $4T$–periodic orbit for $V_{in}=37V$, and in Figure 6.10(d) – Figure 6.10(f) it can be seen a chaotic attractor for $V_{in}=38V$. On the contrary, after applying the adaptive–ramp control, the $1T$–periodic orbits for both cases are obtained, as can be seen in Figure 6.10(g) – Figure 6.10(i) for $V_{in}=37V$, and Figure 6.10(j) – Figure 6.10(l) for $V_{in}=38V$.

Experimental Results

Figure 6-11 shows a complete scheme of the experimental set-up. The ramp waveform can be generated with the circuit proposed in [12]. We have used the component values of Table 6-1.

This implementation is divided into three stages classified as “Buck converter”, “Driver” and “Controller”. The first stage is the system’s plant; in this stage we denote $A$–node as
Figure 6-10: - Numerical phase portrait and time responses for \( V_{ramp}, V_{co} \) and \( V_C \) signals for the system controlled by ramp. (a)-(c) for \( V_{in}=37 \text{V} \) and (d)-(f) for \( V_{in}=38 \text{V} \). - Numerical phase portrait and time responses for \( V_{ar}, V_{co} \) and \( V_C \) signals for the system controlled by adaptive ramp. (g)-(i) for \( V_{in}=37 \text{V} \) and (j)-(l) for \( V_{in}=38 \text{V} \).
the \textit{PWM} power input from the “Driver” stage. This stage has also the \textit{Vco} calculation through a high performance operational amplifier (NE5534) who offers the features such as very low noise, high output-drive capability and low distortion. The second stage is the system’s actuator, where we have used the mosfets IRF540N (\(Q_1\) and \(Q_2\)) like switches (\(S_1\) and \(S_2\) in Figure 6-4 or Figure 6-1). Furthermore, in this step it is necessary to take the low–power control signal (\(u\)) generated by the “\textit{Controller}” stage, and amplify it aiming at increase the power pulse to supply the buck converter. For this purpose we have used the IC IR2112 to drive N–Chanel mosfets as can be seen in the “\textit{Driver}” stage. The IR2112’s input is the \textit{PWM}–control signal within the discrete set \{0,1\}V and the output is the same signal but amplified to the discrete set \{0,4\}V.

Finally, in the “\textit{Controller}” stage we have implemented an analog version of the adaptive ramp as it was explained earlier in this section (using Eq. (6-5)).

As can be seen in Figure 6-12, the experimental phase portrait, and time responses for \(V_{ramp}, V_{ar}, V_{co}\) and \(V_C\) signals are in complete agreement with the diagrams presented in Figure 6-10. The orbits of period grater than one and chaotic behavior have been satisfactory removed using adaptive–ramp control (Figure 6.12(g) – Figure 6.12(l)).
Figure 6-12: Experimental phase portrait and time responses for $V_{ramp}$, $V_{co}$ and $V_C$ signals for the system controlled by ramp. (a)-(c) for $V_{in}=37V$ and (d)-(f) for $V_{in}=38V$. Experimental phase portrait and time responses for $V_{ar}$, $V_{co}$ and $V_C$ signals for the system controlled by adaptive ramp. (g)-(i) for $V_{in}=37V$ and (j)-(l) for $V_{in}=38V$. 
6.3 DC-DC Buck Converter Controlled by Sine Wave

This configuration of the buck converter has been modeled in Chapter 2 and depicted in Figure 2-5.

To perform numerical analysis the component values used in the present section can be seen in Table 6-2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage (V_{in})</td>
<td>(13-70)V</td>
</tr>
<tr>
<td>Reference voltage (V_{ref})</td>
<td>12V</td>
</tr>
<tr>
<td>Inductance (L)</td>
<td>20mH</td>
</tr>
<tr>
<td>Capacitance (C)</td>
<td>47 \mu F</td>
</tr>
<tr>
<td>Load (R)</td>
<td>22\Omega</td>
</tr>
<tr>
<td>Lower voltage (V_{u})</td>
<td>3.8V</td>
</tr>
<tr>
<td>Upper voltage (V_{Lo})</td>
<td>8.2V</td>
</tr>
<tr>
<td>Ramp period (T)</td>
<td>400\mu s</td>
</tr>
<tr>
<td>$A_1$ gain ($a$)</td>
<td>8.4</td>
</tr>
<tr>
<td>$A_2$ gain (Comp)</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

6.3.1 Presence of Chaos in the System

In Figure 6-13 a one–dimensional bifurcation diagram as $V_{in}$ varies is shown, this bifurcation diagram was firstly obtained and analyzed in Section 4.1, where $1T$–periodic orbits, $3T$–, $6T$–, $12T$– and chaos was detected.

With the aim to eliminate the chaotic behavior and orbits of period greater than one, a control technique based on adaptive sine is proposed.

6.4 DC-DC Buck Converter Controlled by Adaptive Sine

In order to counteract the chaos presence, we plan to build a new $T$–periodic sine signal according to the $V_{co}$ behavior and $V_{in}$ variations, as a result, obtain a $T$–periodic adaptive sine ($V_{as}$) which tracks $V_{co}$. The same analysis performed in Section 6.2 is applied to the
buck converter controlled by sine waveform.

As a consequence, the *T–periodic adaptive sine* can be formulated as follows:

\[
V_{as}(t) = \frac{V_{co}}{k} + \frac{V_{in}}{k} \sin \left( \frac{2\pi t}{T} \right)
\]  

(6-6)

where \( V_{as} \) corresponds to the signal which will replace \( V_s \); \( V_{co} \) is the new offset voltage and \( V_{in} \) is the new amplitude. From Eq. (6-6), as in the system controlled by ramp, can be also perceived that any change at any time in \( V_{co} \) and \( V_{in} \) immediately update \( V_{as} \), this allows that \( V_{as} \) changes at the same time \( V_{co} \) changes, achieving that \( V_{as} \) adapts to \( V_{co} \). Note that the term \( k \) makes the adaptive sine waveform \( (V_{as}) \) changes its offset voltage and its amplitude as \( V_{co} \) and \( V_{in} \) vary respectively.

Here we also emphasize that in Section 6.3 or 2.3 the control scheme is based on a fixed sine waveform, meanwhile in this section we are considering the case where the sine waveform is variable. Accordingly, we have simply adjusted the *T–periodic* sine signal to produce a *T–periodic adaptive sine signal* as can be seen in Figure 6-14.
6.4 DC-DC Buck Converter Controlled by Adaptive Sine

Figure 6-14: DC-DC buck converter controlled by adaptive sine.

Summarizing, $V_{co}$ and $u$ are defined by Eqs. (6-7) and (6-8) respectively.

$$V_{co}(t) = a (V_C(t) - V_{ref})$$  \hspace{1cm} (6-7)

$$u = \begin{cases} 1 & \text{if } V_{co} < V_{as} \\ 0 & \text{if } V_{co} > V_{as} \end{cases}$$ \hspace{1cm} (6-8)

### 6.4.1 Obtaining $k$

The objective now is to find the best value for $k$ from Eq. 6-6 and using bifurcation diagrams in the same way than it was done in the system controlled by ramp. Always looking at the values of $k$ and $V_{in}$ that guarantees 1T-periodic solution with good regulation performance.

Firstly, a two-dimensional bifurcation diagram (depicted in Figure 6-15) is computed. $V_{in}$ and $k$ are used as the bifurcation parameters. From Figure 6-15 is observed that the always desired behavior can be achieved for any $V_{in}$ with a properly choice of $k$.

Secondly, it is found the values for which the system exhibits the lowest percentage of regulation error ($\%e$). Figure 6-16 shows the steady state percentage error as $V_{in}$ and $k$ vary.
Figure 6-15: Two-dimensional bifurcation diagram. Bifurcation parameters are $V_{in}$ and $k$. Color codes can be seen at the top of the diagram.

Figure 6-16: Three-dimensional diagram. Bifurcation parameters are $V_{in}$ and $k$, showing the percentage of regulation error ($\%e$). Color codes can be seen at the right side of the diagram.
As can be seen from Figure 6-16, the black dashed line shows the values of $k$ for each value of $V_{in}$ that allow the system to obtain the lowest %e. In this diagram can be also noted that the desired values of $k$ are not always located in the limit of the stability as it happened in the system controlled by ramp.

In Figure 6-17 the lowest percentage error and the stability limit of 1T–periodic orbit for each combination of $V_{in}$ and $k$ are presented; however, the limit of the stability not always coincides with the lowest percentage error. Curves were smoothed by cubic interpolation as well. 1T–periodic orbits taken from Figure 6-17 upper are mapped to 6-17 lower.

![Diagram](image)

**Figure 6-17**: Function that provides suitable values of $k$ according to $V_{in}$.

From Figure 6-15, Figure 6-16 and Figure 6-17 can be observed that it is possible to maintain a 1T–periodic orbit within the $V_{in}$ range [13, 70]V with a properly choice of $k$. Then, in order to prove our control technique numerically it has been computed a bifurcation diagram using Eq. (6-6) and it is depicted in Figure 6-18.

Here it is shown that chaos and orbits of period greater than one were eliminated satisfactorily. Now, comparing this bifurcation diagram with the obtained with a classical sine waveform control (Figure 6-18 and Figure 6-13 respectively), it can be clearly seen that our proposed technique removes successfully the orbits of period greater than one and the chaotic zones; moreover, looking at Figure 6-13, in $V_{in} = 70V$ the system response reaches a maximum value of around $V_C = 15.01V$, hence, obtaining %e with respect to $V_{ref}$ results in 25.08%; whereas looking at Figure 6-18, in $V_{in} = 70V$ the system response reaches a maximum value
of around $V_C = 13.06\, \text{V}$, hence, obtaining $\%e$ with respect to $V_{\text{ref}}$ results in 8.83%; as a result, it can be concluded that $\%e$ has decreased up to 16.25%.

### 6.5 Conclusions

- A new technique for chaos control applied to the ramp–controlled and sine–controlled buck power converter have been proposed.

- The design process of the control scheme is based on adaptive ramp applied to the ramp–controlled system and an adaptive sine applied to the sine–controlled system. Both techniques were explained in detail in this chapter.

- The method of adaptive–ramp control provides a ramp with offset voltage and slope variables that depend on $V_{\text{co}}$; as a consequence, the system can not undergo complex behaviors within the $V_{\text{in}}$ range $[13, 70]\, \text{V}$ with a properly choice of $k$. Similarly, the method of adaptive–sine control provides a sine waveform with offset voltage and amplitude variables that depend on $V_{\text{co}}$; therefore, the system can not undergo complex behaviors within the $V_{\text{in}}$ range $[13, 70]\, \text{V}$ with a properly choice of $k$ too.

- The simplified version of the proposed adaptive–ramp control (with variable offset voltage but fixed slope) is easy to implement, does not need many components and yields to excellent results.
• It was proven numerically and experimentally the capability of the system controlled by adaptive ramp to be robust against broader fluctuations in \( V_{in} \) (within \([20, 40]\text{V})\); as a result, getting rid of all undesired behaviors. In addition, it was also proven numerically the capability of the system controlled by adaptive sine to be robust against broader fluctuations in \( V_{in} \) (within \([13, 70]\text{V})\), canceling all the undesired behaviors as well.

• The nonlinear version of \( k \) is not laborious to implement, but requires more components or it is necessary to use digital devices; for that reason, the designer can select the appropriate implementation or can enlarge even more the \( V_{in} \) range, of the system controlled by adaptive ramp, over which the \( 1T \)-periodic orbit remains stable taking another suitable \( k \) value.

• The system responses showed that the percentage of regulation error decreased up to 7.08\% when the adaptive ramp was working. And decreased up to 16.25\% when the adaptive sine was working.

• The settling time was obtained for the system controlled by ramp and controlled by adaptive ramp as an example. And it can be concluded that the settling time can increase or decrease depending on \( V_{in} \) values. This can be seen in Table 6-3, where \( \text{Settling-time1} \) is the settling time for the system controlled by ramp, and \( \text{Settling-time2} \) is the settling time for the system controlled by adaptive ramp. In other words, the settling time strongly depends on \( V_{in} \) variations.

• Simulation and experimental results were used to show the feasibility of the proposed adaptive–ramp control, concluding that in the DC-DC buck converter it worked perfectly.

### Table 6-3: Settling times for some \( V_{in} \) values obtained from both control strategies.

<table>
<thead>
<tr>
<th>( V_{in} \text{[V]} )</th>
<th>( \text{Settling-time1 [s]} )</th>
<th>( \text{Settling-time2 [s]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0.017</td>
<td>0.01</td>
</tr>
<tr>
<td>26</td>
<td>0.04</td>
<td>0.014</td>
</tr>
<tr>
<td>30</td>
<td>0.011</td>
<td>0.017</td>
</tr>
<tr>
<td>34</td>
<td>0.011</td>
<td>0.024</td>
</tr>
<tr>
<td>36</td>
<td>0.07</td>
<td>0.032</td>
</tr>
<tr>
<td>37</td>
<td>0.022</td>
<td>0.103</td>
</tr>
</tbody>
</table>
7 Future Work

Abstract

In this brief chapter it is suggested some topics that were missing of analyzing or that will continue the path of this research.
• Demonstrate mathematically and experimentally some of the bifurcations found in both systems, the buck converter controlled by ramp and the buck converter controlled by sine wave, in order to support the numerical results.

• Use Lyapunov exponents to provide a qualitative and quantitative characterization of dynamical behavior of the systems. This because detecting and quantifying chaos has become an important task and the spectrum of Lyapunov exponents has proven to be the most useful dynamical diagnostic for chaotic systems.

• Analysis of two-dimensional bifurcation diagrams to understand the complex behavior under the variation of two parameters of the system.

• Develop a general control technique based on bifurcation diagrams for power converters, where the input voltage, the reference and the load would be taken into account in order to provide a very robust response.

• Apply and implement experimentally the afore mentioned control technique to a specific power converter for validating the results.
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