Active Disturbance Rejection Control of Horizontal-Axis Wind Turbines

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Active Disturbance Rejection Control of Horizontal-Axis Wind Turbines

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To my beloved Mother, Father and Brother...

To Chapita Ortíz, and my Cousins James and Marito ;)

Abstract

Wind turbines are complex nonlinear machines whose main objective is to convert the wind energy into electric power. These systems work in two main operating regions. In region 2, the energy captured must be maximized by forcing the turbine speed to proportionally track the wind speed; in region 3, the wind speed is too high and the energy captured must be dissipated and the turbine speed must be regulated to its nominal value. Wind turbines are systems with enormous challenges, not only because of the regulation of speed and power under highly nonlinear aerodynamics, but also due to the high efficiency required even when model uncertainties, periodic disturbances, flexible modes, or system faults are present. This dissertation addresses the control of horizontal-axis wind turbines operating in regions 2 and 3 under the active disturbance rejection control paradigm. New control schemes based on the active disturbance rejection philosophy are proposed in order to tackle three specific problems in wind turbine control, such as: a) wind energy capture maximization of wind turbines operating in region 2, b) regulation of speed and power of wind turbines operating in region 3, and c) reduction of periodic loads on the rotor and the structure of the wind turbine. The proposed schemes are validated using a 5 MW reference nonlinear large-scale wind turbine implemented in the FAST (fatigue, aerodynamics, structures and turbulence) code and tested under realistic 3-D wind speed field. The FAST code is considered as a standard wind turbine dynamic simulation tool in industry. The results showed that the proposed active disturbance rejection control schemes are effective for controlling the wind turbine in regions 2 and 3, with effective attenuation of the periodic load components of the blades.
Table of contents

List of figures xiii
List of tables xvii

1 Introduction 1
  1.1 Background .................................................. 1
  1.2 Wind Turbine Essentials ...................................... 3
  1.3 History and Future of Wind Turbines ......................... 5
  1.4 Wind Turbine Model .......................................... 6
    1.4.1 Aerodynamic model .................................... 8
    1.4.2 Structural model ..................................... 8
    1.4.3 Pitch mechanism and generator models ................. 10
  1.5 Motivation and Problem Statement .......................... 11
    1.5.1 Wind energy capture maximization ..................... 12
    1.5.2 Speed/power regulation in full-load region ......... 12
    1.5.3 Rejection of periodic loads ......................... 12
  1.6 Contributions ............................................... 13
  1.7 List of Publications ........................................ 14
  1.8 Structure of the thesis ..................................... 15

2 Literature review 17
  2.1 Introduction ................................................ 17
  2.2 Standard Control Methods ................................... 19
    2.2.1 Standard Torque Control (STC) ....................... 19
    2.2.2 Maximum Power Point Tracking (MPPT) ............... 20
    2.2.3 Aerodynamic Torque Feedforward (ATF) ............. 21
    2.2.4 Disturbance Accommodating Control (DAC) .......... 21
    2.2.5 Standard Collective Pitch Control ................... 26
Table of contents

2.3 Advanced Control in Region 2 ................................................. 27
2.4 Advanced Control in Region 3 ................................................. 31
2.5 Rejection/Reduction of periodic disturbances ....................... 35
2.6 Discussion ........................................................................... 40
2.7 Conclusions ........................................................................ 41

3 ADRC approach to Maximize Energy Capture in Wind Turbines 43
3.1 Problem formulation ............................................................... 43
3.2 Benchmark Model and Baseline Controller .............................. 44
3.3 Aerodynamic torque estimation via GPI observer .................... 45
  3.3.1 Disturbance internal model and Augmented system .......... 45
  3.3.2 Degree of approximation of the disturbance internal model .. 47
  3.3.3 Observer Design ............................................................... 47
  3.3.4 Results ........................................................................... 48
3.4 GPI Control ........................................................................... 48
  3.4.1 Control Design ................................................................. 50
  3.4.2 Results ........................................................................... 53
  3.4.3 Robustness ..................................................................... 58
3.5 GPI Observer-Based Control .................................................... 58
  3.5.1 Control Design ................................................................. 58
  3.5.2 Zero dynamics ................................................................. 62
  3.5.3 Results ........................................................................... 62
3.6 Conclusions ........................................................................... 67
  3.6.1 ADR/GPI Control ............................................................... 67
  3.6.2 ADR/GPI Observer-based Control ................................. 67

4 ADRC approach of Wind Turbines Operating in Full-Load Region 69
4.1 Introduction ........................................................................... 69
4.2 Wind turbine model ............................................................... 70
  4.2.1 Operating trajectory of the wind turbine ......................... 70
  4.2.2 Open-loop uncertain system model ................................. 71
4.3 Robust ADR Collective Pitch Control Scheme ..................... 75
  4.3.1 Disturbance internal model and augmented system .......... 75
  4.3.2 Disturbance estimation ................................................... 77
  4.3.3 Control scheme ............................................................... 78
  4.3.4 Results ........................................................................... 79
4.4 ADR Individual Pitch Control approach for Periodic Disturbances 86
# Table of contents

4.4.1 System Model for IPC ........................................ 86  
4.4.2 ADR Observer-based Control Scheme ............................. 90  
4.4.3 Spatial ADR Observer-based Control Scheme ...................... 92  
4.4.4 Results ............................................................. 100  
4.5 Conclusions .......................................................... 103  
4.5.1 Robust ADR collective pitch control scheme ...................... 103  
4.5.2 ADR individual pitch control for periodic load reduction .......... 106  

5 Concluding remarks .................................................. 107  

References ........................................................... 109  

Appendix A Proofs ...................................................... 123  
A.1 Proof of Theorem 3.1 ............................................... 123  
A.2 Proof of Theorem 3.2 ............................................... 124  
A.3 Proof of Theorem 3.3 ............................................... 124  
A.4 Proof of Theorem 3.4 ............................................... 125  
A.5 Proof of Theorem 4.1 ............................................... 126  
A.6 Proof of Theorem 4.2 ............................................... 127  
A.7 Proof of Theorem 4.3 ............................................... 127  
A.8 Proof of Theorem 4.4 ............................................... 129
List of figures

1.1 Wind Energy World Total Installed Capacity. ................................. 1
1.2 Structure and components of an horizontal-axis wind turbine. .............. 2
1.3 Components and actuators of a horizontal-axis wind turbine. ............... 3
1.4 Operating regions of a wind turbine. ......................................... 4
1.5 Subsystems interconnection of a wind turbine. ................................ 7
1.6 Curve of power coefficient $C_p$ of a 4.8MW wind turbine. ................ 9
1.7 Two-mass model of a wind turbine. ........................................... 10

2.1 Standard Torque Control. ......................................................... 19
2.2 Standard torque control with adaptive gain. ................................. 20
2.3 Aerodynamic Torque Feedforward control scheme. ........................... 21
2.4 Disturbance tracking control scheme for energy capture maximization. . 24
2.5 Disturbance tracking control plus individual pitch control for energy capture maximization and load reduction. ......................... 25
2.6 Collective pitch control using full-state feedback DAC. .................... 25
2.7 Standard gain scheduling PI control method for wind turbines operating in region 3 ................................................................. 27
2.8 Sliding mode control/observer strategy for wind energy maximization. . 27
2.9 Nonlinear Control of a Variable-Speed Wind Turbine using a Two-Mass Model. 28
2.10 Optimal reference speed scheme based on aerodynamic torque estimation. . 29
2.11 Wind energy maximization using robust control and extremum seeking control. 29
2.12 DAC scheme for IPC and CPC with periodic control/observer gains. .... 32
2.13 Comparison of performance of the LQG controllers. .......................... 33
2.14 Nominal/robust active/passive fault-tolerant LPV controller. .............. 33
2.15 IPC for mitigating wind shear effects. ....................................... 36

3.1 Aerodynamic torque estimation results on a 4.8MW wind turbine. ......... 49
3.2 Optimal angular speed trajectory calculation from aerodynamic torque. .. 49
List of figures

3.3 Anti-windup implementation of the GPI controller. 53
3.4 Closed-loop system scheme of the proposed ADR/GPI control strategy. 53
3.5 Simulation results of aerodynamic torque estimation and optimal trajectory generation. 54
3.6 Simulation results of the proposed ADR/GPI robust control law. 55
3.7 Simulation results of the ADR/GPI Control law on power converter fault. 56
3.8 Closed-loop frequency response using the proposed ADR/GPI control law. 57
3.9 Open-loop frequency response using the proposed ADR/GPI control law. 57
3.10 Closed-loop system scheme of the proposed ADR observer-based control strategy. 63
3.11 Simulation results of the proposed GPI observer-based control. 64
3.12 Simulation results using the proposed ADR/GPI Observer-based control approach on power converter fault. 65
3.13 Performance comparison of the proposed control strategy by switching off the estimation of $\Delta_1(t)$. 66

4.1 Optimal operating trajectory of the wind turbine $(\bar{\omega}_r, \bar{\beta}, \bar{V}_w)$. 71
4.2 Partial derivatives of $F_T$ and $T_r$ evaluated along the optimal operating trajectory. 72
4.3 Comparison of the open-loop responses due to change in wind speed and collective blade pitch for the 5 MW reference wind turbine in FAST code and the obtained uncertain model (4.4). 74
4.4 Eigenvalue location of each ADR CPC scheme for the 64-vertices-polytope system. 80
4.5 Stepwise wind speed profile. 82
4.6 Simulation results comparing the 3 proposed ADR control schemes vs the baseline controller using a wind rise/fall profile. 83
4.7 Class A Kaimal turbulence wind speed profile. 84
4.8 Simulation results of 3 proposed ADR CPC schemes vs the baseline controller under a 25% turbulence intensity wind profile. 85
4.9 General scheme of the IPC and CPC strategies via the Coleman Transform. 87
4.10 Block diagram of the fixed-frame Coleman transformed system. 88
4.11 Block diagram of the wind turbine control system showing the collective pitch control loop and the Coleman transform-based IPC scheme to be designed. 89
4.12 Detailed block diagram of the ADR/IPC Observer-based control scheme 92
4.13 Spectral content of blade bending moments for both rotating and fixed coordinate frame of a 5 MW wind turbine (FAST code) under steady wind speed of 17 m/s. Rated rotor speed: 12.1 rpm, Spatial-sampling: 0.005 rev. First row: spectrum of time-domain signals; Second row: spectrum of spatial-domain signals. .......................................................... 94

4.14 Detailed block diagram of the spatial ADR/IPC control scheme. ............ 100

4.15 Block diagram of the GS/PI collective pitch control with 1P Coleman transform IPC scheme (CPC+MBC1P). .......................................................... 101

4.16 Closed loop simulation results of flap-wise bending moments for the control schemes under different turbulence wind profiles. ......................... 104

4.17 Closed loop simulation results of tilt and yaw bending moments for the control schemes under different turbulence wind profiles. ....................... 105
List of tables

2.1 Advanced control strategies for wind turbines operating in region 2. 30
2.2 Advanced control strategies for wind turbines in region 3. 35
2.3 Advanced control strategies for reduction of periodic disturbances in wind turbines. 39

4.1 Parameters of the 5 MW wind turbine used for the study. 71
4.2 Maximum and minimum values for the linear parameter varying terms of the uncertain model of the 5 MW wind turbine. 72
4.3 LMI Control problems of the proposed ADR control scheme. 79
4.4 List of the design parameters of each ADR collective pitch controller. 81
4.5 Speed and power data analysis for turbulent profile. 85
4.6 Standard deviation of flap-wise bending moments (kNm) for proposed control schemes under different wind profiles. 101
4.7 Standard deviation of hub-tilt bending moments (kNm) for proposed control schemes under different wind profiles. 102
4.8 Standard deviation of hub-yaw bending moments (kNm) for proposed control schemes under different wind profiles. 102
4.9 Standard deviation of generated power (kW) for each control scheme under different wind profiles. 102
Chapter 1

Introduction

1.1 Background

Wind energy has a history of over a hundred years. The technology developed to get benefit from it has risen from the experimental to be nowadays the highest growth rate renewable energy source in the world [1]. In fact, its growth has been exponential as shown in Fig. 1.1 [2].

![Fig. 1.1 Wind Energy World Total Installed Capacity.](image)

Wind turbines are systems that convert wind energy into electric power. There exist two types of wind turbines: horizontal-axis and vertical-axis. Currently, horizontal-axis wind turbines (see Fig. 1.2 [3]) are the most commonly used type mainly due to its remarkable capabilities in energy capture and production. They work at variable-speed combined with both generator torque control and blade-pitch control in order to operate the complete Wind Energy Conversion System (WECS). In this scenario, control systems are needed to properly regulate the extracted power from the wind, since depending on the wind speeds affecting the system, the wind energy captured must be maximized or regulated.
The larger the wind turbines size is the more wind energy can be captured. Therefore, big turbines have economic advantages and this is evidenced by the impressive sizes of the wind turbines today (like a Boeing 747 or a football field [4]). Nevertheless, due to the dimensions of the modern wind turbines and their new construction materials, the whole structure is susceptible to undergo highly flexible modes. This type of behavior can degrade the lifetime of the WECS, increase the maintenance and rise the energy production costs.

![Structure and components of an horizontal-axis wind turbine.](image)

Wind turbines not only show issues associated to vibrations. Problems like wind speed estimation, disturbance tracking, maximization of energy capture, disturbance rejection, speed regulation, robustness over nonlinearities and flexible modes, are some of the most studied in the literature. Such a vast research suggests that applied control strategies could have an important role in the wind turbine behavior. This situation provides a motivation to consider new alternative control strategies that can improve the performance of the wind turbines.

Today, WECSs represent a mature technology with important potential of development in science and engineering. Advanced control systems can contribute towards a reduction in wind energy production costs by means of rising the efficiency, keeping the structural loads at low levels and increasing the lifetime of each component of the WECS.

The origin of modeling and modern control of wind turbines can be attributed to authors like: Liebst, Mattson and Bossanyi [5, 6], who in the 80’s showed that modern techniques relatively fulfilled requirements such as speed regulation and damping of cyclic loads at the same time. Thus, in comparison to traditional PID control, the results given by the state space techniques let treat multiple requirements embedded in the same controller. However, it is with the coming of the 90’s and beginning of 2000 that this topic became an attractive research field. Since then, contributions by Bongers [7–10], Bossanyi [11–13], Balas [14, 15],
Stol [16, 17], Pierce [18], Leithead and Connor [19] and Vihriala [20] served as framework to start solving operating problems of wind turbines.

1.2 Wind Turbine Essentials

The operation of wind turbines is relatively simple. The wind passes through the rotor blades and creates lift and thrust forces that causes the rotor to move. This rotor movement or rotor power is transmitted through the rotor shaft, also known as Low Speed Shaft (LSS), to the gearbox in order to increase the rotation speed in the High Speed Shaft (HSS). Finally, the mechanical power in the HSS is transmitted to a generator where it is transformed into electrical power. Fig. 1.3 shows the parts and actuators of a horizontal-axis wind turbine [21].

![Fig. 1.3 Components and actuators of a horizontal-axis wind turbine.](image)

The wind turbines are equipped with different types of sensors used for purposes of monitoring and in control loops. The position and speed of the rotor are measured by means of encoders. An anemometer is located on the nacelle which is useful to determine wind speed and it is particularly used for determining the start or stop of the wind turbine. Different devices are also used in the wind turbine, such as: sensors to measure the generated-power, strain-gauges on the tower and blades to measure deformations and loads, accelerometers on the tower to measure fore-aft and side-to-side accelerations, encoders for measuring the position and velocity of the blade-pitch, torque transducers on the rotor and generator shafts, etc. [4, 21, 22].

Wind turbines have in general three types of actuators (see Fig. 1.3). The first type of actuator is the Yaw system, which aligns the nacelle to the wind direction. This actuator is
Introduction

not used at high speeds (greater than $1^\circ/s$) because that can trigger dangerous gyroscopic forces, developing torsional modes in the tower. The second type of actuator is the Generator, which can be controlled to follow a desired torque and determines how much torque is extracted from the wind turbine. The third type of actuator is the blade-pitch system, which is responsible for angular positioning (see Fig. 1.3) of each blade individually (Individual Pitch Control (IPC)) or collectively (Collective Pitch Control (CPC)). Varying the pitch-angle of the rotor blades changes the aerodynamic torque generated by the wind. Speed limits of the blade-pitch system vary depending on the size of the wind turbine, for example, $18^\circ/s$ for 600 kW wind turbines and about $10^\circ/s$ for 5 MW wind turbines [21].

Wind turbines have three main regions of operation which are detailed in Fig. 1.4. When the wind speed is low (lower than Cut-in speed), the available power in the wind is low compared to the losses in the wind turbine. In consequence, the wind turbine is stopped and is considered operating in the region 1. Region 2 is a mode of operation when the wind speed is ranging from the cut-in speed to the rated speed, and the main goal is to maximize wind energy capture. In this operating region, the loads on the structure are generally small and the generator torque is often the only control input. The blade-pitch angle $\beta$ is kept constant due to the capture of power is maximized for a particular value of the angle of the blades $\beta_{opt}$ [4, 23].

In region 3, which considers wind speeds greater than rated-speed but less than cut-out speed, the main objective is to keep the generated power at its nominal value. In this region, the captured energy must be limited so that do not exceed safe limits and loads. A popular strategy is to maintain the generated power at its nominal value, let the generator torque be constant and control the wind turbine angular speed to its nominal value by means of a collective control of the blades-pitch angle [23, 24].

For each operating region, the curve of Fig. 1.4 illustrates the relationship between the generated power vs. wind speed on a 2 MW horizontal-axis wind turbine. The region 1 shows that the generated power is zero because the wind turbine is stopped, in region 2 the
generated power increases non-linearly with respect to the wind velocity, and in region 3, the generated power is kept constant at its nominal value (higher magnitudes may overheat the electronic systems of the turbine).

1.3 History and Future of Wind Turbines

The first dependable information of the existence of windmills is dated in 644 A.C. and it is reported as a vertical-axis windmill used by Persians to grind grains. Some centuries after, Chinese people used vertical-axis windmills as a watering system in their rice crops. In Europe, the horizontal-axis windmill was independently invented from the vertical-axis windmill of east. The first verifiable information about windmills was in Normandy in 1180 and spread throughout Europe, Finland and Russia. Also, lots of windmills were found in Germany in the 15th century, the windmill had the rotor entirely of wood and the stone tower [25].

In Holland, great improvements were developed to the windmills by the XVI century. The windmills could rotate to align with the wind direction by means of a basic gear system and brakes. These improvements were perfected until middle of XIX century. In this century, it was introduced the classic American water pumping wind system; the need for this machine was enhanced by the phenomenon of agriculture in the Midwestern United States. More than a million of these systems were sold in the middle east and west in early 1850 [26].

In 1888, Charles F. Brush combined a windmill with a D.C. generator which is known as the first large scale wind turbine for energy production. The wind turbine had a 17 m rotor and 12kW of rated power. In 1891, the Danish physicist and meteorologist Paul la Cour developed a wind turbine with primitive aerodynamic shapes. One of his students named Johannes Juul, built in 1957 the Gedser 200 kW wind turbine, which is seen as one of the greatest achievements in the history of the wind turbines. In 1941 the Smith-Putnam wind turbine, the world’s first megawatt-size wind turbine, was connected to the local electrical distribution system in Vermont, USA. The 1.25 MW turbine operated for 1100 hours before a blade failed [27].

The next big steps occurred due to the oil crisis experienced in 1973. This crisis began a rapid market growth in California (United States) and caused the production of large quantities of low-quality wind turbines in the first generation. This concludes in a poor image of wind technology and the market fell in the late 80’s. In Europe, markets grew from the early 90’s especially in Germany, Denmark and Spain [27]. This growth led to wind turbines increasingly larger and more efficient, ranging from 20-60kW fixed-speed wind turbines in the 80’s, 5 MW variable-speed wind turbines in 2004, to 6 MW, 7.5 and 10 MW
direct-drive generator wind turbines in 2011 [28]. Today, the combination of several wind turbines working together, known as wind farm, facilitates the production of wind energy at a cost as competitive as conventional energy production. Historical details of wind turbines can be found in [25, 26, 29].

In 2011 the UpWind project \(^1\) showed that the design of 20 MW variable-speed horizontal-axis wind turbines is feasible. The project remarked that researches are needed to develop lighter and more rigid rotors. Moreover, UpWind probed that new advanced blade designs could reduce loads on the rotor in 10\% by using more flexible materials, but added that it is necessary to investigate new advanced control strategies and algorithms that can be applied for reducing loads on the rotor [30].

At present, investigations are addressed to study, on one hand, new control possibilities on the rotor blades including structural changes of the turbine; and on the other hand, new control schemes without modifying the structure of the rotor. Under the first direction, in 2008 was developed the concept of smart-rotor to increase the control distribution over each blade of the rotor, and LPV control strategies [31]. In 2009 and 2010 [32–34], in order to provide load reduction over the wind turbine rotor, the performance of both blade-pitch control and several types of active aerodynamic devices including: micro-tabs, morphing edges and conventional flaps, were investigated.

In 2010 and 2011, Lackner and Castaignet [35, 36] researched the load-fatigue reduction capabilities of Trailing Edge Flaps (TEFs) located along of the rotor blades of a wind turbine. Also in 2010 [37], the feasibility of using synthetic jet actuators to enhance the performance of wind turbine blades on a small scale blade model (wind tunnel experiments) was studied. In [38] is presented an experimental and computational study to assess the potential of plasma actuators (electro-fluid devices) to alter the lift of airfoils in view of controlling the output power of wind turbines under high wind speed conditions. In [39], the abilities of load alleviation of a smart rotor equipped with TEFs with MIMO feedback controllers are studied.

The second direction, which considers control schemes without including any structural change on the wind turbine, is addressed in chapter 2. This direction is treated deeper in chapter 2 because this thesis lays within this direction.

1.4 Wind Turbine Model

The first modern contributions about modeling horizontal axis wind turbines can be attributed to Peter Bongers and Gregor van Baars, who in the early 90’s presented a simple flexible model of a wind turbine validated with experimental input-output data [40]. Then, more

\(^1\)http://www.upwind.eu
1.4 Wind Turbine Model

precise models (in low frequency ranges) were developed in which dynamics of tower, blades, gear system, generator, pitch mechanism and aerodynamics were integrated [41]. Nowadays, there are sophisticated models called benchmarks used to validate controllers against realistic wind conditions and model uncertainties. Two popular benchmarks in the field are the one published by Odgaard et al. [42] and the complete FAST (Fatigue, Aerodynamics, Structures, and Turbulence) Code which is a comprehensive aeroelastic simulator capable of predicting both the extreme and fatigue loads of two- and three-bladed horizontal-axis wind turbines (HAWTs) [43]. These two benchmarks are used to validate the proposed control strategies in this work.

In the follows, a nonlinear dynamic model for horizontal axis wind turbines suitable for variable speed control purposes is presented. This model has been reported in several papers related to the modeling and control of WECS [21, 44–47]. This work focuses the research on horizontal-axis variable-speed wind turbines because, although the fixed-speed ones are easy to build and to operate [26], variable-speed ones have up to 20% more energy extraction capability than the fixed-speed version [48]. In addition, variable speed wind turbines are much more complex to control, that is why the role of control systems is of great importance in the performance of large wind turbines.

Below are presented the aerodynamic and structural model of a wind turbine, which includes: the blades aerodynamics, the gear system, the tower, the blade pitch positioning mechanism and a simplified model of the generator. Fig. 1.5 shows the interconnection of each subsystem of the WECS.

![Fig. 1.5 Subsystems interconnection of a wind turbine.](image)
1.4.1 Aerodynamic model

The aerodynamic interaction between the wind and the turbine blades produces a torque $T_r$ which rotates the rotor with angular speed $\omega_r$, and a thrust force $F_T$ acting on the turbine nacelle. The rotor power of the turbine depends on the wind speed relative to the rotor $V_e$, the air density $\rho$, the rotor swept area $S$ and the rotor aerodynamic properties. The wind turbine rotor power and torque, and thrust force of each blade are expressed in terms of non-dimensional power ($C_P$), torque ($C_Q$) and thrust coefficients ($C_T$) as follows [47]:

$$P_r(t) = \frac{1}{2} \rho S C_P(\lambda(t), \beta(t)) V_e^3(t) \quad [W]$$  \hspace{1cm} (1.1)

$$T_r(t) = \frac{1}{2} \rho S C_Q(\lambda(t), \beta(t)) V_e^2(t) \quad [Nm]$$  \hspace{1cm} (1.2)

$$F_T(t) = \frac{1}{2} \rho S C_T(\lambda(t), \beta(t)) V_e^2(t) \quad [N]$$  \hspace{1cm} (1.3)

with $R$ is the radius of the rotor, $\omega_r(t)$ is the rotor speed and $\beta(t)$ is the collective pitch angle of the blades. The torque coefficient is defined as

$$C_Q(\lambda, \beta) = \frac{C_P(\lambda, \beta)}{\lambda} \quad (1.4)$$

where the tip-speed ratio (TSR) is defined as:

$$\lambda(t) = \frac{R \omega_r(t)}{V_e(t)} \quad (1.5)$$

The curves of $C_P$, $C_Q$ and $C_T$ depend on the specific wind turbine being studied. Figure 1.6 shows the $C_P$ curve from the benchmark [42].

1.4.2 Structural model

The structural model consists of the dominant dynamics of the blades, drive-train and the tower. The combined dynamics is defined as [44, 47]:

$$(m_t + N m_b) \ddot{y}_t + N m_b r_b \dddot{\xi} + B_t \dot{y}_t + K_t y_t = N F_T(\lambda, \beta, V_e) \quad (1.6)$$

$$m_b r_b \dddot{\tilde{y}}_t + m_b r_b^2 \dddot{\xi} + B_{b} \dddot{\xi} + K_{b} r_b^2 \dddot{\xi} = r_b F_T(\lambda, \beta, V_e) \quad (1.7)$$
where $m_t$ is the equivalent top mass on the tower, $N$ is the number of the blades, $m_b$ is the equivalent mass of a blade, $r_b$ is the radius of the blade at which the equivalent thrust force $F_T$ is applied, $y_t$ is the fore-aft bending displacement of the tower and $\xi$ is the flapwise angular displacement of the blades. $B_t$ and $B_b$ are the equivalent damping coefficients, and $K_t$ and $K_b$ are the equivalent stiffness coefficients of the tower and blades, respectively. The relative wind speed in the rotor is defined as $V_e = V_w - \dot{y}_t - r_b \dot{\xi}$, where $V_w$ denotes the absolute wind speed.

Consider a two-mass model for the drive-train of the wind turbine (see Fig. 1.7 [49]), then the following dynamic equations for the rotor and generator angular speed, are derived [45]:

$$J_r \ddot{\omega}_r = T_r (\lambda, \beta, V_e) + \frac{B_{dt}}{N_g} \omega_g - K_{dt} \theta_s - (B_{dt} + B_{ls}) \omega_r$$ (1.8)

$$J_g \ddot{\omega}_g = \frac{K_{dt}}{N_g} \theta_s + \frac{B_{dt}}{N_g} \omega_r - \left( \frac{B_{dt}}{N_g^2} + B_{hs} \right) \omega_g - T_g$$ (1.9)

$$\dot{\theta}_s = \omega_r - \frac{1}{N_g} \omega_g$$ (1.10)

where $J_r$ and $J_g$ are the rotor and generator inertia, $B_{hs}, B_{dt}$ and $B_{ls}$ are the equivalent damping coefficients of the high-speed shaft, drive-train and the low-speed shaft, respectively. $K_{dt}$ is the drive-train equivalent stiffness coefficient and $N_g$ is the gear ratio of the transmission. $T_g$
is the generator torque, \( T_r \) is the rotor aerodynamic torque. The torsion angle of the rotor shaft is given by \( \theta_s = \theta_r - \left( \frac{1}{N_g} \right) \theta_g \) with \( \theta_r \) and \( \theta_g \) the rotor and generator angles, respectively.

### 1.4.3 Pitch mechanism and generator models

The pitch mechanism is often modeled as a first order system \([45, 46]\):

\[
\dot{\beta}(t) = -\frac{1}{\tau_\beta} \beta(t) + \frac{1}{\tau_\beta} \beta_d(t) \quad [\text{rad/s}]
\]

(1.11)

where the input \( \beta_d(t) \) is the reference angle. To represent the physical limitations of the pitch actuators, the model may include constraints on the slew rate and the range of the pitch angle.

The power converter and generator dynamics are given by \([42]\).

\[
T_g(t) = -\frac{1}{\tau_g} T_g(t) + \frac{1}{\tau_g} T_{g,d}(t) \quad [Nm/s]
\]

(1.12)

\[
P_g(t) = \eta_g \omega_g(t) T_g(t) \quad [W]
\]

(1.13)

respectively, where \( T_{g,d}(t) \) is the desired generator torque (control input), \( P_g(t) \) is the produced power by the generator, and \( \eta_g \) represents the generator efficiency.
1.5 Motivation and Problem Statement

Large modern wind turbines are machines with enormous challenges, not only because of the regulation of speed and power under highly nonlinear aerodynamics, but also due to the high efficiency required even when model uncertainties, external disturbances, or system faults are present. As a consequence, the efficiency of power capture and power generation is strongly dependent on the selected control method [50] and this represents an important potential of research and development in science and engineering. This situation provides a motivation to consider new alternative control techniques to make the WECS more efficient and reliable.

A large number of control schemes to find the best way of solving the energy capture maximization problem for wind turbines at low-to-medium wind speeds have been proposed (see, e.g., [17–20, 51–54]). The control techniques range from standard torque control [18], disturbance tracking control [15, 17], maximum power point tracking [19], and aerodynamic torque feedforward [20] to complex nonlinear strategies [50–54]. On the other hand, for operating wind turbines in full load region there are two trends: speed/power regulation and load reduction. In speed regulation using collective pitch control, we can find accommodation control techniques [14, 55–57], and robust LTI or LPV controllers [58–60]. In load reduction techniques, we can find Coleman Transform-based schemes, robust $H_{\infty}$-based techniques and repetitive control schemes, see e.g. [61–66], among others are already proposed.

Most of these techniques deal with the wind turbine complexity using linearization techniques or nonlinear control. Following a different approach, some of the active-disturbance-rejection- (ADR-) based techniques allow linear control solutions for some class of uncertain complex nonlinear systems and can offer a linear, simpler, and robust solution to the wind turbine operating problems. This is the case of the ADR philosophy-based technique called generalized proportional integral (GPI) Control [67] and its GPI observer-based control extensions [68, 69]. Generalized proportional integral (GPI) control technique was started in 2000 by Fliess et al. [67, 70] and involves in its design the active rejection of disturbances. The dual counterpart of the generalized proportional integral controller, called GPI observer, was introduced in [69], in the context of sliding mode observers for flexible robotics systems. The nonsliding version appears in [68] applied to chaotic systems synchronization. The GPI control strategies have been adapted, extended, and applied successfully in areas other than wind energy, such as induction motor control [71], chaotic systems control [72], and power converters control [73].

GPI observer-based control of nonlinear uncertain systems is very much related to methodologies known as disturbance accommodation control (DAC) [74] and active disturbance rejection control (ADRC) [75–77]. In this control paradigm, disturbances, unmodeled dynamics and parameter uncertainty are treated as a lumped disturbance signal. This uni-
fied disturbance signal is estimated on-line with a predefined level of approximation and then is canceled by a control law that is remarkably simple by making use of this estimate. This methodology had not been yet explored, extended nor adapted in the wind turbine control field. In this way, this thesis makes contributions by proposing novel, simple and robust solutions to the main open problems of horizontal-axis variable-speed variable-pitch wind turbines, such as: wind energy capture maximization in region 2 (partial-load region), speed/power regulation in region 3 (full-load region), and reduction of rotor loads.

1.5.1 Wind energy capture maximization

When wind turbines are operating in region 2, i.e. wind speeds are in low-to-medium range, the main objective is to maximize the wind energy capture. In other words, to operate the wind turbine at its maximum aerodynamic efficiency, the power coefficient $C_P(\lambda, \beta)$ must stay in its optimal point $C_{P\text{opt}} = C_P(\lambda_{\text{opt}}, \beta_{\text{opt}})$. As a consequence, the generator speed $\omega_g$ must be controlled to track $\omega_{g\text{opt}}(t) = \frac{N_g \lambda_{\text{opt}}}{R} V_w(t)$. From this, two main issues can be named: wind speed estimation and power loss due to uncertainty in $\lambda_{\text{opt}}$.

1.5.2 Speed/power regulation in full-load region

This operating zone involves high wind speeds above the rated wind speed. The wind turbine must limit the captured wind energy through modification of the blade pitch angle such that mechanical loads keep low and at the same time the generator angular speed and the produced power are regulated to their nominal values. In this region, the relation from the blade pitch angle to the aerodynamic torque is highly nonlinear. Even more, when there is a constant wind profile the wind turbine model changes periodically as a function of the rotor position [78]. The application of control techniques in this region is nowadays an active research field.

1.5.3 Rejection of periodic loads

Wind turbines are influenced mainly by two effects termed wind shear and tower shadow. The term wind shear is used to describe the variation of wind speed with height, while the term tower shadow describes the redirection of wind due to the tower structure [79, 80]. In three-bladed turbines, the most common and largest periodic disturbances occur at what is known as 1P, 2P and 3P frequencies (this is one, two and three times the rotor frequency). Thus, even for a constant wind speed at a particular height, a turbine blade would encounter variable wind as it rotates. Blade load moments are observed due to the periodic variations
of wind speed experienced at different locations [81]. Additionally, such periodic variations in the aerodynamic torque contribute significantly decreasing the life-time of each blade due to fatigue accumulation [82].

1.6 Contributions

This thesis presents contributions to both the ADR Control theory and the Wind turbine control field. These contributions are summarized as follows:

**ADRC design methods for wind turbines operating in region 2:** In region 2, the design problem can be stated as finding a controller that maximizes power capture and provides robustness against disturbances without measuring the wind speed. Under this idea, two new ADR control designs are proposed based on the estimations provided by a GPI aerodynamic torque observer. In the first design, a robust GPI controller is proposed in which the optimal rotor speed trajectory is tracked by using high order disturbance derivatives. In the second design, a dual GPI-observer-based control law is proposed to estimate and reject system nonlinearities, uncertainties and other disturbances (including actuator faults).

**ADRC design methods for speed control of wind turbines operating in region 3:** In the conventional ADRC design, with the aim of maintaining a simple tuning, repeated observer poles are placed in order to adjust the observer’s bandwidth, but this option is not an adequate tuning, especially in high order systems or with lightly damped modes. Here a pole placement LMI-based technique is used to handle this issue in ADRC. ADRC is traditionally and conceptually based on a simplified model of the system, realized by means of a chain of integrators. However, under this approach the unified equivalent input disturbance takes into account a large quantity of endogenous dynamics, and therefore, the extended observer requires more responsibility given the greater uncertainty of the model. LPV models emerge as a linear modeling possibility with great capabilities of resembling the nonlinear behavior of a system; then, a controller, designed using a more specialized model, is expected to provide better performance and robustness indexes. Here, a robust LPV GPI observer based control scheme is designed (from LPV models) to solve the problem of controlling the speed and electric power of a large variable-pitch wind turbine operating in full-load region affected by high turbulence intensity.

**ADRC design methods for periodic load reduction in wind turbines:** Control of periodic loads on blades of a wind turbine is an open problem today. This work proposed two
new low-order ADR control approaches to effectively address periodic load reduction in wind turbines operating in full load region. The proposed approaches tackle the load reduction problem based on the ADR-philosophy by means of two observer-based schemes: an ADR/IPC resonant observer-based control technique and an ADR/IPC spatial-domain resonant observer-based control technique.

1.7 List of Publications


1.8 Structure of the thesis

This thesis starts with an introductory chapter that presents the background and basics of wind turbine control, history and future of WECSs, motivation and contributions. Then, chapter 2 summarizes and discusses the state of the art of each addressed problem. In chapter 3, an aerodynamic torque observer is proposed and two new ADR control schemes are proposed and evaluated to address the problem of wind energy capture maximization. Chapter 4 presents three new ADR control approaches for wind turbines operating in full-load region: the first one addresses the problem of speed/power regulation in region 3, and then two ADR schemes are proposed to reduce periodic load moments on the rotor blades. Finally, conclusions and future work are outlined in Chapter 5.


Chapter 2

Literature review

2.1 Introduction

Wind turbines have many control levels, they are called: supervision, operational and subsystems. The supervision level monitors the structural health of the wind turbine and determines when to start it or stop it. The operational level determines how the wind turbine accomplishes its control objectives in regions 2 and 3. In the subsystems level there are controllers in charge of commanding each actuator of the wind turbine, such as: generator torque control, yaw control system and blade-pitch control.

From control engineering point of view, a wind turbine is a machine with great challenges due to multiple causes: it works by means of a stochastic input, exhibits highly non-linear dynamics, suffers vibrations and structural loads, is affected by periodic disturbances, among others. Accordingly, the wind turbine behavior is highly affected by the employed control strategy, and this motivates the development and validation of new alternative control algorithms that can get the wind turbine performance improved. That’s why in the last 20 years, several authors have proposed a lot of control strategies for the operation of horizontal-axis wind turbines.

Until recently, with some exceptions, industrial wind turbines used PID control systems tuned by trial and error. Moreover, to avoid stability problems by flexible modes of wind turbines, its components and elements were fabricated using quite rigid materials [83]. Today it is known that due to interactions between the structure and the wind turbine, almost all flexible modes are excited during normal operation, and damping these modes using control techniques is increasingly critical as the size of the wind turbine increases [55].

Classical control techniques like PID control were used in some wind turbines between the 70’s and 80’s. Many of them were fixed-speed wind turbines built with very rigid transmission systems and large inertia in the rotor. It was found that the first torsional
mode of the transmission system was affected by wind turbulence, and consequently the control objectives were regulate power and add damping to this mode using blade-pitch control. In these studies, it was found that system dynamic response was closely related to the bandwidth of the controller, and it was determined that a large bandwidth introduces the possibility of exciting multiple modes of vibration of the turbine, such as torsional modes in the transmission system and the first vibration mode of the tower [6].

In the 80’s, modern control methods using state space representations were an alternative to attack multiple control objectives in view of the knowledge of the states of the system. In 1983, Liebst applied LQG Control to a NASA 100kW wind turbine to treat effects as wind-shear and tower-shadow [5]. However, this study did not consider other flexible modes of the wind turbine besides blade flap-wise. At that time, modern methods successfully met the control objectives compared to conventional PID control; the results were promising due to the treatment of multiple requirements of a wind turbine. In 1984 Mattson reported the use of state observers to estimate the wind speed, and Bossanyi in 1987 reported an adaptive control method, consisting of a state-estimator with variable gains to improve energy capture [6].

In the 90’s, modern control of wind turbines became an attractive research field that led to lots of publications. Peter Bongers, in the early 90’s introduced LQG and $H_{\infty}$ controller applications for wind turbines operating in region 3; the first one, in order to reduce loads on the turbine [7], and the second to maintain stability under system uncertainties [8]. Bongers investigated the ability of $H_{\infty}$ control to reduce loads on the rotor shaft despite of the uncertainties in the model [9] and then used multiple $H_{\infty}$ controllers to operate regions 2 and 3 of a wind turbine [84]. Mark Balas in 1996 presented the first application of Disturbance Accommodating Control (DAC) to a wind turbine [14], and in 1998 presented the theory of Disturbance Tracking Control (DTC) for wind turbines [15].

In the 90’s and early 2000, contributions by Bongers [7–10], Bossanyi [11–13], Balas and Stol [14–16] served as framework to start solving operating problems in each region of a wind turbine. These researches led to the development of known control strategies such as: Disturbance Tracking Control (DTC) [15, 17] in 1998, the well known Standard Torque Control (STC) in 1999 [18], Maximum Power Point Tracking (MPPT) Control in 2000 [19], Aerodynamic Torque Feedforward (ATF) Control in 2001 [20] and Disturbance Accommodation Control (DAC) [14, 62] first time applied in 1996.

Since 2009, it can be found researches on DAC [55–57], nonlinear control strategies [85, 86], robust LPV control [46, 87, 88], robust fault-tolerant control [45, 89], repetitive control [63–65], super-twisting controllers/observers to maximize energy capture [48, 52, 53,
90, 91], among others. Some excellent reviews and tutorials on wind turbine control can also be detailed in [4, 21, 22, 24, 92].

This chapter seeks to synthesize the work of many researchers in the field of onshore horizontal-axis wind turbine control. We bring together and classify the most pertinent findings of a large number of studies. This chapter also provides interpretative analysis and a historical perspective of the wind turbine control field.

2.2 Standard Control Methods

In this section, the widely used control methods for the operation of wind turbines in regions 2 and 3 are detailed. Challenges in the standard control methods are highlighted.

2.2.1 Standard Torque Control (STC)

The standard control scheme for variable speed wind turbines operating in region 2 is showed in Fig. 2.1. In this strategy the blade pitch angle is fixed to its optimum value $\beta_{opt}$ and the generator torque control law is governed by

$$T_{g,ref} = k \omega_r^2,$$  \hspace{1cm} (2.1)

$$k = \frac{1}{2} \rho \pi R^5 \frac{C_{p_{max}}}{\lambda_{opt}^3},$$  \hspace{1cm} (2.2)

where $k$ is the optimum control gain [21, 22]. This widely used strategy for energy capture has some disadvantages that can result in deficiencies of energy capture [93]. First, the control gain is difficult to determine because of the dependence of a very accurate model ($C_{p_{max}}$ and $\lambda_{opt}$), particularly because blade aerodynamics can significantly change over time [90].
Second, the standard value of $k$ could not provide maximum energy capture under real-world conditions. Johnson et al. [94] showed via simulation that smaller values of $k$ may result in improved energy capture dependent on the wind turbulence intensity, and then an adaptive control scheme which improved energy capture in the presence of parametric uncertainty [22] was proposed. The Fig. 2.2 shows the adaptive control scheme, where the air density $\rho$ is assumed measured on-line and the parameter $M$ is adapted according to the average power.

\[
M(k) = M(k-1) + \Delta M(k)
\]
\[
\Delta M(k) = \gamma_{\omega} \, \text{sgn}[\Delta P_m(k)] \, \text{sgn}[\Delta P_{\text{ref}}(k)]
\]

\[
M(k) = \rho \omega^2 + \Delta M(k)
\]

\[
\Delta M(k) = \gamma_{\omega} \, \text{sgn}[\Delta P_m(k)] \, \text{sgn}[\Delta P_{\text{ref}}(k)]
\]

\[
\frac{1}{2} \rho \pi R^2 C_{\text{max}} V_w^3
\]

**Fig. 2.2 Standard torque control with adaptive gain.**

### 2.2.2 Maximum Power Point Tracking (MPPT)

Wind energy, although plentiful, varies continuously as the wind speed changes throughout the day. The amount of generated power of a wind turbine depends on the precision with which the peak power is followed by the control system. MPPT algorithms maintain the operating point of the wind turbine around this peak, this is known as Maximum Power Point Tracking [95]. The MPPT algorithms can be classified into three main control methods: Tip-Speed-Ratio Control (TSRC), Power Signal Feedback Control (PSFC) and Hill-Climb Search Control (HCSC) [96]. The method TSRC regulates generator speed to keep $\lambda$ at its optimum value $\lambda_{\text{opt}}$, in which the extracted power is the maximum. This method requires the measurement or estimation of wind speed and generator speed, it is also necessary to know the optimum value of $\lambda$ [97].

It is possible to build a power control loop using the rotor measured speed in conjunction with the measured power and an internal torque control loop. The desired power is defined by

\[
P_{\text{opt}} = \frac{1}{2} \rho \pi R^2 C_{\text{max}} V_w^3
\]

so that, when the power tracking error is zero, the operating point of the wind turbine is maintained around the maximum power point, this is known as PSFC [95]. One drawback of
2.2.3 Aerodynamic Torque Feedforward (ATF)

Based on the feedback of rotor/generator speed, it can be constructed a torque control law using (2.1), known as standard torque control. A variant of this control law, known as Aerodynamic Torque Feedforward, was shown in [20], where the aerodynamic torque and generator speed are estimated using a Kalman filter and then injected into the control law. The ATF control scheme is described in Fig. 2.3 [86]. A disadvantage of this control structure is that the rotor speed can present high variations [95], and the strategy does not cancel the steady-state error [86].

\[
\hat{\beta}_{opt} = \frac{1}{2} \rho \pi R^5 \frac{C_{p_{max}}}{\lambda_{opt}}
\]

\[
\dot{\phi}_g = \frac{B_g}{N^2 g} + \frac{B_y}{N^2 g} \dot{\phi}_g + \frac{1}{N g} T_{g,ref} - T_g,
\]

Fig. 2.3 Aerodynamic Torque Feedforward control scheme.

2.2.4 Disturbance Accommodating Control (DAC)

The theory of Disturbance Accommodating Control was proposed in [74, 98]. This theory was first applied to the control of wind turbines in 1996 [14]. Since then, several publications have applied and extended this approach. Below, a review of the DAC theory is performed, including its three basic variations [55].

Consider the MIMO system:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Fw(t) \\
y(t) &= Cx(t),
\end{align*}
\]
where \( w(t) \) is a vector of \( p \) uncertain disturbances affecting the states of the system according to the matrix \( F \). If the disturbances \( w(t) \) can be modeled or at least approximated by an homogeneous differential equation, then the theory of DAC can be used. There are three main ways of disturbance accommodating: Disturbance Cancellation Control (DCC), Disturbance Minimization Control (DMC), and Disturbance Utilization Control (DUC).

The DCC technique, seeks to eliminate all the effects of \( w(t) \) in the state vector \( x(t) \), designing \( u(t) \) such that:
\[
Bu(t) \equiv -Fw(t), \quad \forall t.
\]

(2.5)

When it is not possible to satisfy the condition (2.5), there still exists for every \( t \), a control law \( u(t) \) that minimizes the term \( \|Bu(t) + Fw(t)\| \). Then, the control law \( u(t) \) is called Disturbance Minimization Control. The Disturbance Utilization Control technique seeks the best way to utilize the energy in the disturbances \( w(t) \) in order to help achieve the control objectives. This mode of disturbance accommodation reduces the energy applied by the control law while the negative effects thereof are mitigated.

Suppose the system (2.4) is completely controllable and observable. Let be the control input \( u(t) \) a two part function \( u(t) = u_s(t) + u_d(t) \), then replacing \( u(t) \) into (2.4) results:
\[
\dot{x}(t) = (Ax(t) + Bu_s(t)) + (Bu_d(t) + Fw(t)).
\]

(2.6)

The control input \( u_d(t) \) is designed to cancel the disturbance, that is \( Bu_d(t) \equiv -Fw(t), \quad \forall t \), and \( u_s(t) \) is designed as a state feedback control law. If \( u_d(t) \) is chosen such that \( Bu_d(t) = -F\hat{w}(t) \), where \( \hat{w}(t) \) is a precise estimation of \( w(t) \), then all effects of \( w(t) \) are canceled. This is known as disturbance cancellation. However, disturbance cancellation requires that \( rank([B \quad FH]) = rank(B) \), otherwise, disturbance cancellation is not possible and disturbance minimization should be applied.

When disturbance cancellation is not possible, there still exists the possibility of finding \( u_d(t) \), such that:
\[
\min_{u_d} \|Bu_d(t) + FH\hat{w}(t)\|.
\]

(2.7)

The solution to the problem (2.7) is unique and is given by \( u_d(t) = -B^\dagger FH\hat{w}(t) \), where \( B^\dagger \) denotes the pseudoinverse of \( B \). This solution always gives the best possible minimization of \( w(t) \) with respect to the Euclidean norm.

Under these schemes, Balas et al. published a DAC-based method named disturbance tracking control (DTC) to maximize energy capture in region 2 [15, 17]; Stol and Balas applied DAC with periodic matrices [16] to control a wind turbine in region 3; and Wright and Balas [99] applied DAC to a 600kW wind turbine operating in region 3.
2.2 Standard Control Methods

used DAC controllers to mitigate cyclic loads on the blades induced by the interaction wind turbine - Vortex effect. Also, an IPC scheme was applied for the same 600kW wind turbine in region 3 using DAC by means of periodic matrices [101]. Also in [102] a DAC scheme was used for mitigating fatigue loads in the blades of a wind turbine operating in region 3. Under the DUC approach, the excess of wind energy is not completely rejected by the controller but utilized in the best way. An application of DUC to a wind turbine operating in region 3 was reported in [55–57]. In order to apply these techniques to wind turbines, linear models or linear periodic models around an operating point or trajectory must be obtained.

Disturbance Tracking Control (DTC)

Disturbance tracking control is a technique developed to operate wind turbines in region 2 [15]. Given the plant model:

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_d V_w \\
y &= Cx
\end{align*}
\]

where, the internal model of the disturbance \( u_d \) is given by:

\[
\begin{align*}
\dot{z}_d &= Fz_d \\
V_w &= Hz_d.
\end{align*}
\]

Then, if

1. \((A, B)\) is controllable

2. \(\bar{A} = \begin{bmatrix} A & B_d H \\ 0 & F \end{bmatrix}, \bar{C} = \begin{bmatrix} C \\ 0 \end{bmatrix}\) is observable

3. \(QH = CL\)

\(\begin{bmatrix} (A + BG_x) & -LF + BG_T + B_d H = 0 \end{bmatrix}\)

the tracking error, \(e_y = y - QV_w \xrightarrow{t \to \infty} 0\), and therefore, disturbance tracking is produced by using the following control law:

\[ u = G_x \hat{x} + G_T \hat{z}_d \]

with \(\hat{x}\) and \(\hat{z}_d\) given by the following observers:

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + Bu + B_d \hat{V}_w + K_x (y - \hat{y}) \\
\dot{\hat{y}} &= C\hat{x} \\
\dot{\hat{z}}_d &= F\hat{z}_d + K_d (y - \hat{y}) \\
\dot{\hat{V}}_w &= H\hat{z}_d
\end{align*}
\]
where, $K = [ K_x \ K_d ]^T$ is chosen such that eigenvalues of the matrix $\bar{A} - K \bar{C}$ have desired locations, $G_x$ is chosen such that $(A + BG_x)$ has desired eigenvalues and $G_T$ is calculated using the condition 3. The Fig. 2.4 shows the block diagram of the DTC technique, where $G = [ G_x \ G_T ]$ and $\bar{B} = [ B \ 0 ]^T$.

Fig. 2.4 Disturbance tracking control scheme for energy capture maximization.

Under the same approach, DTC has been combined with IPC in order to provide blade load mitigation [17]. Thus, the control input is defined as:

$$Bu = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with, $u_1 = \Delta T_{g,ref}$, $u_2 = \Delta \beta_{ref}$, $\Delta \omega_r = T x$. Then, the control law is defined as:

$$\begin{cases} u_1 = G_1 \Delta \omega_r + G_T z_d \\ u_2 = G_2 \omega_r \end{cases}$$

The Fig. 2.5 shows the control structure with:

$$G = \begin{bmatrix} G_1 T & G_T \\ G_2 & 0 \end{bmatrix}, K = \begin{bmatrix} K_x \\ K_d \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ 0 & 0 \end{bmatrix}.$$
2.2 Standard Control Methods

Disturbance Accommodating Control

Following the same approach, DAC can be applied to wind turbines operating in region 3. A full-state feedback control of a WT is presented using both periodic and constant control gains [16]:

\[
\Delta \dot{x} = A \Delta x + B \Delta \beta + B_d \Delta V_w, \quad \Delta \beta = G_f \Delta x + G_{df} \Delta V_w
\]

\[
G_f = -\frac{1}{R} B^T P, \quad G_f(t) = -\frac{1}{R} B(t)^T P(t)
\]

\[
G_{df} = -B_d ^T, \quad G_{df}(t) = -B(t)^T B_d(t)
\]

where \(G_f(t)\) and \(G_{df}(t)\) are periodic control gains, \(R\) and \(P\) are design parameters obtained from a LQ tuning technique. The Fig. 2.6 shows the control scheme, which is easy to note some disadvantages such as assuming that the wind speed is measured as well as all the states of the wind turbine.
Continuing his own work, Stol et al. [61] based on DAC theory presents an observer based controller with periodic control gains in order to regulate the rotor speed and reduce cyclic loads on the rotor blades for a wind turbine operating in region 3. The system model, the control law and the periodic control gains are as follows:

\[
\begin{align*}
\Delta \dot{\mathbf{x}} &= A(t) \Delta \mathbf{x} + B(t) \Delta \beta + B_d(t) \Delta V_w, \\
\Delta \beta &= G_x(t) \Delta \dot{\mathbf{x}} + G_d(t) \Delta \dot{\mathbf{z}}_d, \\
G_x(t) &= -\frac{1}{R} \mathcal{B}(t)^T P(t), \\
G_d(t) &= -\mathcal{B}(t)^\dagger \mathcal{B}_d(t),
\end{align*}
\]

where, \(\Delta \dot{\mathbf{x}}\) and \(\Delta \dot{\mathbf{z}}_d\) are provided by the following state estimator:

\[
\begin{align*}
\Delta \dot{\mathbf{z}} &= \tilde{A}(t) \Delta \mathbf{z} + \tilde{B}(t) \Delta \beta + \mathcal{K}(t) (\Delta \mathbf{y} - \Delta \mathbf{\hat{y}}), \\
\Delta \mathbf{\hat{y}} &= \mathcal{C}(t) \Delta \mathbf{\hat{z}},
\end{align*}
\]

with,

\[
\begin{align*}
\tilde{A}(t) &= \begin{bmatrix} A(t) & B_d(t) \\ 0 & 0 \end{bmatrix}, & \tilde{B}(t) &= \begin{bmatrix} B(t) \\ 0 \end{bmatrix}, & \mathbf{\hat{z}} &= \begin{bmatrix} \mathbf{\hat{x}} \\ \mathbf{\hat{z}}_d \end{bmatrix}, \\
\mathcal{C} &= \begin{bmatrix} C & 0 \end{bmatrix}, & \mathcal{K}(t) &= \begin{bmatrix} K_x(t) & K_d(t) \end{bmatrix}^T.
\end{align*}
\]

### 2.2.5 Standard Collective Pitch Control

A gain scheduling PI control is the industrial standard for collective pitch control of wind turbines operating in region 3. The baseline collective pitch Gain Scheduling (GS) PI controller is defined as [78]:

\[
\Delta \beta_d(t) = K_P(\beta) N_g \omega_r(t) + K_I(\beta) \int_0^t N_g \omega_r(t) dt, \tag{2.8}
\]

with,

\[
\begin{align*}
K_P(\beta) &= \frac{2 \left( J_r + N_g^2 J_g \right) \omega_N \omega_p \omega_{pi}}{N_g \left( \frac{\partial P_r}{\partial \beta} \right)}, \tag{2.9} \\
K_I(\beta) &= \frac{\left( J_r + N_g^2 J_g \right) \omega_N \omega_{pi}^2}{N_g \left( \frac{\partial P_r}{\partial \beta} \right)}. \tag{2.10}
\end{align*}
\]
where, $\zeta_{pi}$ is the desired damping ratio, $\omega_{pi}$ rad/s is the desired natural frequency and $\partial P_r/\partial \beta$ is the partial derivative of the rotor power $P_r$ respect to the collective blade pitch angle $\beta$ in watt/rad, which is scheduled by means of a look-up table. The Fig. 2.7 shows the standard control method for wind turbines operating in region 3. Note that this control law schedules the control gains by means of $\beta$.

Fig. 2.7 Standard gain scheduling PI control method for wind turbines operating in region 3.

2.3 Advanced Control in Region 2

Beltran et al. [90], showed that in the STC the captured power is assumed equal to $T_g \omega_r$, and thus the term $(J_r + N_g^2 J_g) \dot{\omega}_r + (B_r + N_g^2 B_g) \omega_r$ is neglected from the dynamic equation of the rotor (see equations (1.8)-(1.10)), so it is reasonable to think that in many cases, and particularly for turbulent winds, this assumption is not realistic. Under this scenario, Beltran et al. proposed a control strategy to tackle this problem based on the estimation of the aerodynamic torque using a high order sliding mode observer, and a high order sliding mode controller. The strategy used by Beltran et al. is shown in Fig. 2.8.

Fig. 2.8 Sliding mode control/observer strategy for wind energy maximization.
Boukhezzar et al. [86], shows the standard torque control strategy adapted for a two-mass mechanical model of a wind turbine, where the torque of the generator is defined by

\[
T_{g,\text{ref}} = k_{\text{opt, hss}} \omega_g^2 - k_{\text{hss}} \omega_g, \tag{2.11}
\]

\[
k_{\text{opt, hss}} = \frac{1}{2} \rho \pi R^5 \frac{C_{\text{p, max}}}{N_g^3 \lambda_{\text{opt}}} \tag{2.12}
\]

\[
B_{\text{hss}} = \left( B_r + \frac{B_r}{N_g^2} \right), \tag{2.13}
\]

where \( B_{\text{hss}} \) is the damping ratio of the LSS seen from the HSS. This approach is described in [86] as a MPPT strategy taken from [19]. Boukhezzar et al. points out two drawbacks of this strategy: (a) the transitions caused by high wind speed fluctuations lead to significant power losses, and (b) many of the dynamic aspects of the wind turbine are not taken into consideration. Then, a dynamic nonlinear state feedback controller is proposed providing slightly better results. Boukhezzar and Siguerdidjane [86], proposed nonlinear static and dynamic state-feedback controllers, based on a two-mass (to include flexible modes of the rotor shaft) and an estimator of the wind speed, for controlling a wind turbine operating in region 2 (see Fig. 2.9). The main objective of the proposed controllers was to optimize the capture of wind energy while the loads in the rotor shaft and the gearbox system are limited. The results showed that the nonlinear dynamic state-feedback controller provided better performance in the presence of disturbances and measurement noise when compared to Standard Torque Control (STC) and Maximum Power Point Tracking (MPPT) methods.

Fig. 2.9 Nonlinear Control of a Variable-Speed Wind Turbine using a Two-Mass Model.
Under a similar idea, Rocha [3] presented a control strategy for the operation of a wind turbine in region 2, which removes the direct measurement of the wind speed. As shown in Fig. 2.10, the estimated aerodynamic torque was used for determining an optimal reference speed to maximize energy capture (the same methodology of ATF), and a discrete-time LQG/LTR controller was designed to maximize wind energy capture and minimize the loads on the turbine.

Fig. 2.10 Optimal reference speed scheme based on aerodynamic torque estimation.

Hawkins et al. [103], proposed a control strategy which consisted of a nonlinear robust controller in conjunction with an extremum seeking controller (see Fig. 2.11). The robust strategy, controls the rotor speed and estimates the aerodynamic torque. The other controller, uses the method of Extremum Seeking Gradient Search and the estimation of the aerodynamic torque to update the TSR and the desired angle of the rotor blades. The controller showed to be robust under moderate wind turbulence and measurement noise.

Fig. 2.11 Wind energy maximization using robust control and extremum seeking control.
<table>
<thead>
<tr>
<th>Control method</th>
<th>Reference</th>
<th>Control signals</th>
<th>Objectives/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2/H_{\infty}$ Control</td>
<td>[104]</td>
<td>Torque</td>
<td>Optimum power curve tracking and load reduction on the rotor shaft.</td>
</tr>
<tr>
<td>LPV/Gain Scheduling Control</td>
<td>[105]</td>
<td>Torque</td>
<td>Maximization of energy capture, damping of resonant modes of the rotor shaft, robust stability against high frequency uncertainties.</td>
</tr>
<tr>
<td>STC with adaptive gain</td>
<td>[22]</td>
<td>Torque</td>
<td>Maximization of energy capture, rotor speed asymptotically stable only when the wind speed is constant.</td>
</tr>
<tr>
<td>STC+Nonlinear robust control</td>
<td>[106, 107]</td>
<td>Torque, IPC</td>
<td>Maximize the energy produced and reduce the mechanical stresses only when the maximum tolerable load limits are exceeded.</td>
</tr>
<tr>
<td>PI+Nonlinear state-feedback control</td>
<td>[108]</td>
<td>Torque</td>
<td>Optimize energy capture while avoiding strong transient responses on the gearbox system.</td>
</tr>
<tr>
<td>Lyapunov-based control</td>
<td>[109]</td>
<td>Torque, CPC</td>
<td>Optimize the power capture with unknown parameters $C_{P_{\text{max}}}$ and $\lambda_{\text{opt}}$.</td>
</tr>
<tr>
<td>LQG, Nonlinear state feedback control</td>
<td>[85]</td>
<td>Torque</td>
<td>Maximize wind power capture (track the optimum TSR), reduce loads submitted by the drive train shaft.</td>
</tr>
<tr>
<td>LQG/LTR</td>
<td>[3]</td>
<td>Torque</td>
<td>Maximize wind energy capture and minimize the loads on the LSS. The estimated aerodynamic torque was used for determining an optimal reference speed.</td>
</tr>
<tr>
<td>Second order SMC and SMO</td>
<td>[90]</td>
<td>Torque</td>
<td>Maximize wind energy capture and robustness.</td>
</tr>
<tr>
<td>Nonlinear static and dynamic state-feedback control</td>
<td>[86]</td>
<td>Torque</td>
<td>Optimize the capture of wind energy while the loads in the rotor shaft and the gearbox system are reduced. Wind speed is estimated with the Newton Algorithm.</td>
</tr>
<tr>
<td>Nonlinear robust control with Extremum seeking control</td>
<td>[103]</td>
<td>Torque, CPC</td>
<td>Optimize the power capture with unknown parameters $C_{P_{\text{max}}}$ and $\lambda_{\text{opt}}$.</td>
</tr>
<tr>
<td>Model-assisted ADRC</td>
<td>[110]</td>
<td>Torque</td>
<td>Optimize energy by tracking $\omega_{\text{opt}} = V_w \lambda_{\text{opt}}/R$, wind speed is assumed measured.</td>
</tr>
</tbody>
</table>

Table 2.1 Advanced control strategies for wind turbines operating in region 2.
The Table 2.1 summarizes other relevant works of wind turbine control in partial-load region. In general, the review shows that:

- In order to avoid wind speed measurement, many papers tend to use the estimate of the aerodynamic torque to calculate the optimal reference of the rotor/generator speed.

- The most common method to estimate the optimal reference speed is to use a kalman observer to estimate the aerodynamic torque $\hat{T}_r$ and then apply

$$\omega_{r_{opt}} = \frac{\lambda_{opt}}{R} \sqrt{\frac{2\lambda_{opt}}{\rho ARC_{P_{max}}} \hat{T}_r}.$$

- The Newton-Rapson algorithm is the most used technique to estimate the wind speed.

- Once the wind speed is estimated or assumed measured through LIDAR sensors, the optimal reference speed is calculated using

$$\omega_{r_{opt}} = \frac{\lambda_{opt}}{R} \hat{V}_w.$$

- The LSS flexible mode is commonly damped using an additional feedback loop based on the generator speed measurement. The active drive train damping is deployed by adding a signal to the generator torque command to compensate for the oscillations in the drive train. This signal should have a frequency equal to the eigenfrequency of the drive train.

- The tight tracking of the maximum $C_P$ will lead to high mechanical stress and thus, transfer aerodynamic fluctuations into the power system. Therefore, it will result in less energy capture.

- In order to achieve a compromise between energy capture improvement and dynamic loads reduction, an intermediate tracking error $e_{\omega_r}(t) = \omega_r(t) - \omega_{r_{opt}}(t)$ dynamics should be chosen.

- Besides disturbance tracking control which is based on DAC, the active disturbance rejection control schemes are few used to control wind turbines in region 2.

- Higher-order disturbance estimations have not been explored.

2.4 Advanced Control in Region 3

Stol [101] evaluated the performance of periodic DAC with IPC and CPC on the two-bladed Controls Advanced Research Turbine (CART) located in Colorado (USA). Fig. 2.12 shows...
the periodic DAC structure. The controller design was based on periodic state-space models and optimal control methods were used for calculating the periodic control/observer gains. The performance of the controllers was verified and compared with a baseline controller regarding the capabilities to reduce loads on the CART. The results suggested that CPC is more suitable for load reduction in region 2, and IPC is more suitable for load reduction in region 3.

Nourdine et al. [111, 112] investigated the effects of the reduction of fatigue loads and power regulation in four different controllers using LQG+IPC in a wind turbine operating in the region 3. The controllers were designed taking into account different vibration modes of the wind turbine, starting from a completely rigid model to a model with vibrational modes in the gear-train, tower and rotor blades. The results, as shown in Fig. 2.13, indicated a significant reduction of fatigue loads especially in the gearbox and rotor blades when all flexibility modes were taking into account in the control design. The cost function for the LQGi controller was defined as follows, with \( i = 1, \ldots, 4 \):

\[
J_i = \int_0^{\infty} \left( q_{p_i} T^2_{1,2,3,4} + q_{d_i} T^2_{2,3,4} + q_{F_i} T^2_{3,4} + q_{b_1} T^2_{B1} + q_{b_2} T^2_{B2} + r_c T^2_{g,ref} + r_{\beta_1} T^2_{\beta,ref1} + r_{\beta_2} T^2_{\beta,ref2} \right). 
\]

In [45] four LPV controllers for a wind turbine operating in the region 3 are investigated under a low-pressure fault of the pitch system (see the control structure in Fig. 2.14). The controllers are: a nominal LPV controller, an active fault-tolerant LPV controller, a passive fault-tolerant LPV controller, and a robust LPV controller. The simulations showed that both
the nominal and robust LPV controller have very similar performances, however the robust one is preferred because it ensures stability and robust performance for the nonlinear model.

The Table 2.2 summarizes several relevant works of control of wind turbines operating in full-load region. The review shows that:

- The CPC scheme is mainly used to regulate the speed of the wind turbine with the generator torque set to the nominal value.
- The torque control and the IPC scheme are used to damp the drive-train mode and attenuate both the tower fore-aft and the blade flap-wise vibrations.
- Great attention has been taken in linear-parameter-varying, gain scheduling and robust control techniques for operating wind turbines in region 3 due to the nonlinearities and uncertainty of the system.
- Linear matrix inequalities are used as a tool to specify multiple requirements of the control system such as: power/speed regulation, torsional load reduction of LSS, blade bending moments mitigation and actuator usage.
• DAC techniques have been applied using different variations, such as: SISO, MIMO, full-state feedback, periodic/fixated observer gains, periodic/fixated control gains, LQG or LQR tuning framework.

• DAC schemes use linearized models without accounting the uncertainty of the system.

• Active disturbance rejection control schemes have not been explored for operating wind turbines in region 3 considering the disturbances and uncertainties of the system.

• Fault tolerant control schemes are emerging as a trend to control wind turbines operating in region 2 and 3.

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ref.</th>
<th>Control</th>
<th>Objectives/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAC with Observer using periodic and fixed gains</td>
<td>[101]</td>
<td>CPC, IPC</td>
<td>Speed regulation, reduce fatigue damage in tower fore-aft bending, LSS torsion and blade flap-wise bending moments.</td>
</tr>
<tr>
<td>H∞ control with state feedback</td>
<td>[113]</td>
<td>CPC</td>
<td>Turbine speed regulation, reduce vibration and torsional loads on the gearbox system.</td>
</tr>
<tr>
<td>Nonlinear dynamic state feedback torque control + proportional pitch control</td>
<td>[114]</td>
<td>Torque, CPC</td>
<td>Power and speed regulation.</td>
</tr>
<tr>
<td>H∞ MIMO control</td>
<td>[115]</td>
<td>CPC, IPC</td>
<td>Speed regulation, increase damping of the first tower bending mode and reduce 1p fluctuations in blade flap-wise bending moments.</td>
</tr>
<tr>
<td>MIMO LPV control</td>
<td>[87]</td>
<td>Torque, CPC</td>
<td>Energy maximization in region 2 and Power/Speed regulation in region 3.</td>
</tr>
<tr>
<td>Robust MIMO LMI-based control</td>
<td>[116]</td>
<td>Torque, CPC</td>
<td>Covers regions 2 and 3, reduce stress on the drive train and actuator usage, include parameter variations in the design procedure.</td>
</tr>
<tr>
<td>SMC+PI control</td>
<td>[117]</td>
<td>CPC</td>
<td>Speed/power regulation and take into account the saturation in rate and magnitude of blade pitch angles.</td>
</tr>
<tr>
<td>Feedback linearization control with extended Kalman filter</td>
<td>[118]</td>
<td>CPC</td>
<td>Speed regulation only around the rated speed, reductions in fatigue loads in the LLS.</td>
</tr>
<tr>
<td>LQG</td>
<td>[111, 112]</td>
<td>Torque, IPC</td>
<td>Speed/power regulation, damp the first drive-train mode, tower fore-aft and blade flap vibration.</td>
</tr>
<tr>
<td>Digital RST control</td>
<td>[119]</td>
<td>CPC</td>
<td>Speed/power regulation with parameter uncertainty tolerance.</td>
</tr>
</tbody>
</table>

Table 2.2 – Continued on next page
2.5 Rejection/Reduction of periodic disturbances

In large-scale wind turbines, wind turbulence, wind gusts, gravity, wind shear and tower shadow, make the effective forces on the blades vary considerably, seriously affecting the operation of wind turbines. These forces result in significant fatigue loads and vibrations in the rotor blades. Big wind turbines are mainly disturbed by two effects named wind shear and tower shadow. The term wind shear is used to describe the variation of wind speed with height, while the term tower shadow describes the redirection of wind due to the tower structure [79]. Thus, even for a constant wind speed at a particular height, a turbine blade would encounter variable wind as it rotates. Torque pulsations are observed due to the periodic variations of wind speed experienced at different locations [81]. Additionally, such periodic variations in the aerodynamic torque contribute significantly decreasing the life-time of each blade due to fatigue accumulation [82]. These variations lead to 1P (once per revolution), 2P and 4P large components in the blade loads (rotating frame of reference), and 0P and 3P load components on the fixed structure (non-rotating frame of reference) such as the nacelle and tower [58]. This has motivated the development of blade Individual Pitch Control (IPC) methodologies, many of which employ the Coleman transformation (or MBC transformation) to simplify the controller design process [66].

In IPC design, the Coleman transformation expresses the states, inputs and outputs of the nonlinear wind turbine model in a non-rotating coordinate frame. The MBC transformation does not directly result in an LTI system, but the MBC approach usually yields a model that

---

**Table 2.2 Advanced control strategies for wind turbines in region 3.**

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ref.</th>
<th>Control</th>
<th>Objectives/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal/Robust, Active/Passive fault-tolerant LPV control</td>
<td>[45]</td>
<td>CPC</td>
<td>Speed/power regulation, add robustness and tolerance under pitch system fault.</td>
</tr>
<tr>
<td>LIDAR-based adaptive FX-RLS feedforward control</td>
<td>[120], [121]</td>
<td>CPC</td>
<td>Speed/power regulation, reduce both the tower loads and the bending moments of the rotor blades.</td>
</tr>
<tr>
<td>Nonlinear adaptive passivity-based individual pitch control</td>
<td>[122]</td>
<td>IPC</td>
<td>Speed/Power regulation for wind turbines operating in region 3. The inclusion of gradient based adaptation laws allows for the on-line compensation of variations in the aerodynamic torque.</td>
</tr>
<tr>
<td>LPV Control with anti-windup for pitch actuators</td>
<td>[123]</td>
<td>CPC</td>
<td>Speed/power regulation especially in the presence of sudden wind gusts. The design method guarantees closed-loop stability and an optimal $H_\infty$ norm performance while it decreases the pitch activity.</td>
</tr>
</tbody>
</table>
is weakly periodic and averaging of system matrices can result in a LTI model of sufficient accuracy [124].

![Diagram](image)

**Fig. 2.15 IPC for mitigating wind shear effects.**

One of the first works for mitigating the periodic disturbances caused by wind shear without using the MBC transformation, shows a partial feedback linearization applied to each blade pitch angle [125]. According to the flap-wise deflection dynamics of each blade defined in (2.14), the strategy consisted in canceling out the wind shear effect by injecting a periodic term in the control law assuming perfect knowledge of the wind speed over each blade.

\[
I_x\ddot{x} + B_x\dot{x} + K_x x \cos(\beta) + K_1 x = K_2 f_{x1} - K_2 K_1 f_{x2} \cos(\theta_r) - K_2 f_{x3} \beta + K_{yy} \sin(\beta) .
\] (2.14)

The Fig. 2.15 shows the control law of the proposed scheme. This scheme has several drawbacks like, the assumption of wind speed measurement and a completely dependence of the model which causes high sensitivity of a phase correction factor $\delta$.

Since then, several control schemes have been proposed in order to handle the periodic disturbances induced on the rotor blades and the fixed structure of the turbine. Some of them are stated in the rotating reference frame (no transformation) and others in the fixed reference frame through the Coleman transform. Table 2.3 summarizes the most relevant works of control strategies to reduce or mitigate loads on the rotor and the structure of horizontal-axis wind turbines.
<table>
<thead>
<tr>
<th>Control method</th>
<th>Ref.</th>
<th>Type</th>
<th>Objectives</th>
<th>Ref. frame</th>
<th>Assumptions</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback linearization</td>
<td>[125]</td>
<td>IPC</td>
<td>Reduce cyclic fatigue loads on the blades caused by wind shear and effects of gravity.</td>
<td>Rotating</td>
<td>Wind speed measurement</td>
<td>Highly dependent on a phase correction factor.</td>
</tr>
<tr>
<td>Periodic DAC with periodic state estimation</td>
<td>[61]</td>
<td>IPC</td>
<td>Regulate the rotor speed while mitigating cyclic loads on the blades</td>
<td>Rotating</td>
<td>Control/Observer use periodic gains with fixed time period.</td>
<td>Periodic model is valid in steady-state, internal model provides rejection of constant signals.</td>
</tr>
<tr>
<td>DAC</td>
<td>[62]</td>
<td>Torque, CPC, IPC</td>
<td>Regulate rotor speed, damp the flexible modes of the gear-box and reduce blade flap-wise deflections caused by wind shear.</td>
<td>Rotating</td>
<td>Internal model with fixed 0P and 1P frequencies.</td>
<td>Internal model is effective in low-turbulence profiles.</td>
</tr>
<tr>
<td>RST control</td>
<td>[126]</td>
<td>CPC</td>
<td>Speed regulation and avoid the 1P, 2P, 4P and 5P action of the blades.</td>
<td>Rotating</td>
<td>Internal model with fixed 1P, 2P, 4P and 5P frequencies.</td>
<td>Internal model scheme is designed to avoid 1P,...,5P frequencies.</td>
</tr>
<tr>
<td>MIMO LQG with feedforward disturbance rejection control</td>
<td>[127]</td>
<td>IPC</td>
<td>Minimization of the rotor tilt and yaw moments, reject low frequency components on the rotor moments.</td>
<td>Non-rotating MBC1P</td>
<td>Wind estimation based on random walk model.</td>
<td>The internal model does not provide adequate rejection of 2P and 4P components.</td>
</tr>
<tr>
<td>DAC with 1P resonant observer</td>
<td>[128]</td>
<td>IPC</td>
<td>Regulate turbine speed, mitigate the effects of shear across the rotor disk, damp the tower’s first fore-aft mode.</td>
<td>Rotating</td>
<td>Internal model with fixed 1P frequency.</td>
<td>Internal model is effective in low-turbulence profiles.</td>
</tr>
<tr>
<td>$H_{\infty}$ control based on disturbance models</td>
<td>[124]</td>
<td>IPC</td>
<td>Investigate the effects of disturbance model augmentation of $H_{\infty}$ control in MBC framework</td>
<td>Non-rotating MBC1P</td>
<td>Oscillations are modeled as output disturbances of fixed known frequencies and magnitudes</td>
<td>Load reduction is negligible even under low turbulent wind conditions.</td>
</tr>
<tr>
<td>Proportional resonant control</td>
<td>[129]</td>
<td>IPC</td>
<td>Reduce blade bending moments, tilt and yaw moments.</td>
<td>Rotating $\alpha\beta$</td>
<td>Resonant elements use 1P, 2P and 4P fixed frequencies.</td>
<td>Resonant elements are not accurate under medium-to-high wind turbulence.</td>
</tr>
</tbody>
</table>

Table 2.3 – Continued on next page
<table>
<thead>
<tr>
<th>Control method</th>
<th>Ref.</th>
<th>Type</th>
<th>Objectives</th>
<th>Ref. frame</th>
<th>Assumptions</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI resonant control</td>
<td>[130]</td>
<td>IPC</td>
<td>Reduce blade bending moments, tilt and yaw moments including unbalanced loads.</td>
<td>Non-rotating</td>
<td>Resonant elements use 1P and 3P fixed frequencies.</td>
<td>Resonant elements are not accurate under medium-to-high wind turbulence.</td>
</tr>
<tr>
<td>PI Control</td>
<td>[131]</td>
<td>IPC</td>
<td>Reduce blade bending moments, tilt and yaw moments.</td>
<td>Non-rotating</td>
<td>Three Coleman transform control loops with six dynamic MBC transformations.</td>
<td>To many control loops, do not compensate for unbalanced loads.</td>
</tr>
<tr>
<td>Nonlinear blade vibration damper with a lead compensator</td>
<td>[132]</td>
<td>IPC</td>
<td>Mitigate blade fatigue loads.</td>
<td>Rotating</td>
<td>The damper is used to damp the modes of the blades, which is designed by nonlinear dynamic inversion method to deal with the non-affine non-linearity of the WT model.</td>
<td>There is no explicit way to tune and select the frequency component or components to attenuate.</td>
</tr>
<tr>
<td>Collective pitch nonlinear ADRC with periodic pitch strategy</td>
<td>[133]</td>
<td>CPC, IPC</td>
<td>Speed regulation and mitigate blade flap-wise periodic loads.</td>
<td>Rotating</td>
<td>Periodic IPC is a cancellation assuming exact model. ADRC is not designed to assure closed-loop stability for all operating points.</td>
<td>IPC is only effective in low-turbulence and for 1P frequency.</td>
</tr>
<tr>
<td>Sliding mode control</td>
<td>[134]</td>
<td>IPC</td>
<td>Reduce asymmetric loads on the blades.</td>
<td>Non-rotating</td>
<td>Controller designed to mitigate the 0P frequency. The scheme added robustness against system uncertainty.</td>
<td>The scheme does not consider attenuation of higher frequencies or a method to select the frequency component to mitigate.</td>
</tr>
<tr>
<td>Lifted repetitive control (RC)</td>
<td>[63], [64]</td>
<td>IPC, smart rotor</td>
<td>Reject periodic load disturbances on the rotor.</td>
<td>Rotating</td>
<td>Multiple memory loops in the lifted RC to robustify the performance against period mismatch, MIMO lifted RC tuned with LQG solution.</td>
<td>The lifted RC is robust to small (1%) changes in period time. The controller results in a complex and high-order model.</td>
</tr>
</tbody>
</table>

Table 2.3 – Continued on next page
Table 2.3 Advanced control strategies for reduction of periodic disturbances in wind turbines.

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ref.</th>
<th>Type</th>
<th>Objectives</th>
<th>Ref. frame</th>
<th>Assumptions</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subspace predictive repetitive control</td>
<td>[65]</td>
<td>IPC</td>
<td>Asymptotically suppress the dominant periodic loads in the wind turbine.</td>
<td>Non-rotating 1P and 2P</td>
<td>Repetitive control strategy with on-line subspace identification. The system parameters are assumed to vary slowly with time.</td>
<td>When the system parameters also vary with time, no statement can be made regarding the stability of the adaptive control law.</td>
</tr>
</tbody>
</table>

The review shows that:

- The vast majority of the proposed schemes are mainly focused on reducing the 1P and 2P frequency components of the blade loads. Activity of the blade pitch system should be limited up-to the 4P component.

- All proposed schemes use control approaches that depend on disturbance internal models assuming that the frequency components of the blade loads 1P, 2P, etc. are invariant, however those frequencies change with the speed of the rotor. Therefore the internal models are accurate under a small variation of the rotor speed.

- There is no active disturbance rejection control scheme applied to mitigate the frequency components of the blade loads.

- There are control schemes, such as proportional resonant control and PI resonant control, composed by fixed resonant elements in the controller. The effect in disturbance rejection of these resonant elements should be explored by including the resonant terms into an extended state observer.

- The use of high order controllers such as repetitive controllers in order to mitigate only the first 3 or 4 frequencies of the blade loads, seems to be oversized. In addition, lifted repetitive control results in a complex and high-order scheme.

- Because disturbance internal models and resonant terms are only accurate about 1% of variation of resonant frequency. Control strategies to handle variations in the frequency components of the blade loads should be extensively explored.
2.6 Discussion

A vast quantity of research for the control of variable-speed variable-pitch horizontal-axis wind turbines operating in region 2 and 3 can be found in the literature. In region 2, the problem of maximization of energy capture has been tackled in two main ways. In the first strategy, it is desired to track the optimum TSR towards the $C_{\text{P,max}}$ by means of tracking the optimum rotor/generator speed which can be calculated using either the wind speed estimation/measurement or the estimate of the aerodynamic torque. This strategy is subjected to accurate estimation/measurement of either wind speed or rotor torque, and the control law must be tuned to avoid tight tracking of $C_{\text{P,max}}$, which will lead to high mechanical stress and thus, it will result in less energy capture. In this point, an intermediate tracking error dynamics should be chosen, but a precise tuning technique is still needed in order to achieve a compromise between energy capture improvement and dynamic loads reduction. In the second strategy, much less explored but with potential benefits, an optimum power reference is obtained using the wind speed (estimation or measurement), and a control law is designed to track the optimum power by commanding the generator torque. When the control law uses the relative degree of the system, an unstable zero dynamics results which deserves to be examined in detail. Also, a tight tracking of the optimum power will lead to high mechanical stress with less energy capture, and thus a precise tuning methodology should be formulated.

The development and application of active disturbance rejection control schemes for operating wind turbines in region 2 has been minimum. One of the applied control schemes which is based on DAC is known as disturbance tracking control. The review shows that in this control scheme a poor approximation of the disturbance ($\dot{V}_w = 0$) is frequently used, also it does not account for system uncertainties in the design, it does not account for disturbances other than wind speed, and in some cases it uses two observers in order to estimate the system states and the wind speed. In addition, under the ADRC scheme it is known that higher-order disturbance estimations are possible, therefore these estimations are available but not used in the control law.

In region 3, the problem is focused on regulating the generator speed and the generated power to their nominal values while the loads of the rotor and structure are alleviated. The regulation of speed is frequently addressed using both robust and LPV control via CPC, and in order to reduce structural loads an IPC technique is added. In this region, the active disturbance rejection control techniques are few used, only DAC techniques have been applied using different variations, such as: SISO, MIMO, full-state feedback, periodic/fixed observer gains, and periodic/fixed control gains. In DAC, linearized models are used without accounting the uncertainty of the system and other disturbances. Under this scenario, active
disturbance rejection control schemes should be adapted to consider uncertainties of the system by means of robust and gain-scheduling LPV tuning approaches.

The increasing dimensions of wind turbines lead to the increase of the loads on wind turbine structure. Pitch control loops can be used to reduce fore-aft tower moments and flap-wise bending moments on the blades, in particular some moments of periodic nature. Some known techniques such as repetitive control and resonant control are effective for rejection of periodic disturbances under precise knowledge of the frequency of the periodic signal described in its disturbance internal model. In wind turbine control, PI resonant control was proposed \([129, 130]\) and also lifted repetitive control was developed \([63, 64]\), both to reject periodic load disturbances on the rotor blades. However, wind turbines are exposed to large disturbances, in consequence the fundamental frequency of the periodic disturbance changes with the rotor speed and those internal models become inexact. Repetitive control was applied under the scheme of lifted models \([63–65]\), in which the results proved to be very acceptable under small variations of the disturbance frequency and therefore, it is suggested an spatial-domain technique to keep the fundamental frequency of the periodic disturbances invariant. In addition, a repetitive control scheme with a high-order internal model used for rejecting two or three components of a periodic signal could be oversized. Instead of using a high order periodic model, control techniques should be focused on the important components to reject, i.e. 1P, 2P, 3P and 4P. That is why, resonant control or a resonant-based control scheme should be deeply explored to provide effectiveness through low-order internal models.

On the other hand, Coleman transform-based IPC techniques proved to be acceptable for periodic load reduction, however in order to reject each frequency of the disturbance, it is necessary to add another MBC control loop. In this sense, it may be necessary to adapt and evaluate new control schemes that use fewer MBC control loops or eliminate the use of MBC control loops without losing design simplicity.

### 2.7 Conclusions

In this chapter the operating regions of horizontal-axis wind turbines and their associated control objectives were reviewed. In this way, the main operating problems of wind turbines and control techniques and strategies for energy capture maximization, speed regulation and periodic load mitigation were mainly revisited. The basic control loops and subsystems of a wind turbine were discussed, then some history from classic PID control schemes to state-space control approaches were outlined. After that, a review of some important control applications and control schemes were classified in three subsections and summarized. Wind
turbine control has been extensively studied, but there still are control schemes that can open up paradigms for advanced control approaches. Various techniques such as active disturbance rejection control, resonant control, or repetitive control have yet to be fully explored and could be adapted and extended to provide new contributions to the wind turbine control field.
Chapter 3

ADRC approach to Maximize Energy Capture in Wind Turbines

3.1 Problem formulation

The following assumptions are stated in relation to the system (1.1)-(1.13):

1. All the parameters of the WECS are known.

2. The pair \((A_{wt}, C_{wt})\) is completely observable.

3. The generator angular speed \(\omega_g(t)\) as well as the generator torque \(T_g(t)\) are available to be used in the control system.

4. For a sufficiently large positive integer \(p\), the disturbance input \(T_r(t)\) exhibits uniformly absolute bounded time derivative of order \(p\). This condition assures the existence of an unknown but finite constant, \(K_{T_r}\), such that

\[
\sup_{t \geq 0} |T_r^{(p)}(t)| \leq K_{T_r}.
\]

For a partial load-operating regime (region 2), the main control objective is the maximization of wind power capture. This objective has a strong relation with the wind turbine power coefficient curve \(C_P(\lambda, \beta)\), which has a unique maximum point that corresponds to the optimal capture of the wind power:

\[
C_P(\lambda_{opt}, \beta_{opt}) = C_{P_{opt}} \quad (3.1)
\]
where
\[ \lambda_{\text{opt}} = \frac{\omega_{g_{\text{opt}}}(t)R}{N_g V_e(t)}. \]  
(3.2)

Accordingly, in order to maximize wind power capture, the blade pitch angle \( \beta \) is fixed to its optimal value \( \beta_{\text{opt}} \), and in order to maintain \( \lambda \) at its optimal value \( \lambda_{\text{opt}} \), the generator speed must be adjusted to track the optimal reference \( \omega_{g_{\text{opt}}}(t) \), given by
\[ \omega_{g_{\text{opt}}}(t) = \frac{N_g \lambda_{\text{opt}}}{R} V_e(t). \]  
(3.3)

Then, it is desired to force the output \( \omega_g(t) \) to accurately track the given trajectory \( \omega_{g_{\text{opt}}}(t) \), independently of the aerodynamic torque input and possible unmodeled disturbance inputs in the WECS, using the desired generator torque \( T_{g,d}(t) \) as the control input and the generator angular speed \( \omega_g(t) \) as the feedback signal.

### 3.2 Benchmark Model and Baseline Controller

The simulations are carried out using a benchmark model for wind turbine control implemented in MATLAB/Simulink. This benchmark model was published by Odgaard et al. \[42\] and can be used to evaluate both fault tolerant and classic control schemes in any region of operation of a wind turbine. The test bench model is based on a realistic nonlinear generic three-bladed horizontal-axis variable-speed wind turbine, containing sensors, actuators, system faults, tower shadow and wind shear effects, full converter coupling, and rated power at 4.8MW.

For wind speeds between 0 and 12.5m/s, the turbine is controlled to operate in region 2. The wind profile used has an average hub-height wind speed of 8.68m/s and a turbulence intensity of 12%. The test bench defines a standard torque control strategy for the operation in region 2 with the following control law:
\[ T_{g,d}(t) = \frac{\eta_{d\tau} \rho \pi R^5 C_{p_{\text{opt}}}}{2 N_g^3 \lambda_{\text{opt}}^3} \omega_g^2(t) = k_{\text{opt}} \omega_g^2(t) \]  
(3.4)

with,
\[ k_{\text{opt}} = 1.2171 \]  
(3.5)
\[ C_{p_{\text{opt}}} = 0.4554 \quad \lambda_{\text{opt}} = 8.0 \quad N_g = 95 \]  
(3.6)
\[ \eta_{d\tau} = 0.97 \quad \rho = 1.225 \quad R = 57.5 \]  
(3.7)
The converter model has the following constraints: max torque gradient $1.25 \times 10^4$ N-m/s, min torque gradient $-1.25 \times 10^4$ N-m/s, max torque $3.6 \times 10^4$ N-m, and min torque 0 N-m.

### 3.3 Aerodynamic torque estimation via GPI observer

In order to obtain the optimal reference $\omega_{g_{opt}}(t)$, the equations (1.2), (1.4), and (3.3) are combined. Therefore, the wind velocity $V_e(t)$ is easily represented as a function of $T_r(t)$ and $C_P$ using (1.2) and (1.4):

$$V_e(t) = \sqrt{\frac{2\lambda T_r(t)}{\rho \pi R^3 C_P(\lambda, \beta)}}.$$  \hfill (3.8)

Then, by replacing (3.8) in (3.3), setting $\lambda$ and $C_P$ to its optimal values, and changing $T_r(t)$ to its estimated version $\hat{T}_r(t)$, the following expression is obtained for the optimal reference trajectory:

$$\hat{\omega}_{g_{opt}}(t) = \frac{N_g \lambda_{opt}}{R} \sqrt{\frac{2\lambda_{opt} \hat{T}_r(t)}{\rho \pi R^3 C_{P_{opt}}}}.$$  \hfill (3.9)

According to (3.9), it is necessary to estimate the aerodynamic torque $T_r(t)$. For that purpose, an extended Luenberger-like linear observer is developed, here referred as GPI observer. The proposed observer uses an approximated internal model of the unknown input disturbance to compose an augmented model for the plant and the disturbance input. Inherent to this kind of observer, a state estimation is also provided. This estimation will be used in the controller design stage in order to track $\omega_{g_{opt}}(t)$.

#### 3.3.1 Disturbance internal model and Augmented system

Given a positive integer $p$, the unknown input disturbance $T_r(t)$ can be modeled by the approximation of its internal model given by

$$\frac{d^p T_r(t)}{dt^p} \approx 0.$$  \hfill (3.10)

Consider the following disturbance states, related to (3.10):

$$x_d(t) = \begin{bmatrix} T_r(t) & T_r(t) & \cdots & T_r^{(p-2)}(t) & T_r^{(p-1)}(t) \end{bmatrix}^T.$$  \hfill (3.11)
where its corresponding dynamics is given by

\[
\frac{d}{dt} x_d(t) = A_d x_d(t) + B_d T_r^{(p)}(t)
\]

\[T_r(t) = C_d x_d(t)\]  

(3.12)

with

\[A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}\]  

(3.13)

where \(x_d(t) \in \mathbb{R}^{p \times 1}\), \(A_d \in \mathbb{R}^{p \times p}\), \(B_d \in \mathbb{R}^{p \times 1}\), and \(C_d \in \mathbb{R}^{1 \times p}\).

The mechanical system (1.8)-(1.10) of the wind turbine can be expressed as:

\[
\frac{d}{dt} x_{wt}(t) = A_{wt} x_{wt}(t) + B_{wt} T_g(t) + F_{wt} T_r(t)
\]

\[y(t) = C_{wt} x_{wt}(t)\]  

(3.14)

with

\[A_{wt} = \begin{bmatrix} -\frac{(B_{dr} + B_{hr})}{J_r} & \frac{B_{dr}}{J_g N_g} & \frac{K_{hr}}{J_r} \\ \frac{B_{dr}}{J_g} & \frac{B_{dr}}{J_g N_g} + B_{hr} & 1 \\ 1 & -\frac{1}{J_g} & 0 \end{bmatrix}, \quad B_{wt} = \begin{bmatrix} 0 \\ -\frac{1}{J_g} \\ 0 \end{bmatrix}, \quad F_{wt} = \begin{bmatrix} \frac{1}{J_r} \\ 0 \\ 0 \end{bmatrix}, \quad C_{wt} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad x_{wt}(t) = \begin{bmatrix} \omega_r(t) \\ \omega_g(t) \\ \theta_s(t) \end{bmatrix}\]

Now, the disturbance states \(x_d(t)\) can be added to the system state vector \(x_{wt}(t)\) to form the following augmented system:

\[
\frac{d}{dt} x(t) = Ax(t) + BT_g(t) + B_a T_r^{(p)}(t)
\]

\[y(t) = Cx(t)\]  

(3.15)
3.3 Aerodynamic torque estimation via GPI observer

with

\[
x(t) = \begin{bmatrix} x_{wt}(t) \\ x_{d}(t) \end{bmatrix}, \quad A = \begin{bmatrix} A_{wt} & F_{wt}C_d \\ 0 & A_d \end{bmatrix}, \quad B = \begin{bmatrix} B_{wt} \\ 0 \end{bmatrix}, \quad B_a = \begin{bmatrix} 0 \\ B_d \end{bmatrix}, \quad C = \begin{bmatrix} C_{wt} & 0 \end{bmatrix}
\]

(3.16)

where \(x(t) \in \mathbb{R}^{(p+3) \times 1}, \ A \in \mathbb{R}^{(p+3) \times (p+3)}, \ B, B_a \in \mathbb{R}^{(p+3) \times 1}, \) and \(C \in \mathbb{R}^{1 \times (p+3)}\).

The next step is to design a GPI observer for the composite system in (3.15) regarding the approximated internal model given in (3.10). The estimated augmented state vector \(x(t)\) contains a real-time estimate of \(x_d(t)\), which is used along with \(C_d\) to recover \(T_r(t)\).

### 3.3.2 Degree of approximation of the disturbance internal model

It is usual to select a first-order approximation mainly because the disturbances are locally taken as additive constant signals or model uncertainties/nonlinearities. In this way, several authors have applied first order disturbance model approximation to different areas, for example: C.D. Johnson used it to model wind speed disturbances in a wind turbine operating in region 3 [135], L. Freidovich and H. Khalil [136] used it to estimate the model uncertainty and disturbance on a nonlinear system, Z. Gao and S. Zhao also used a first order internal model disturbance approximation to estimate the resonance in two-inertia systems [137].

Nevertheless, the disturbance internal model in (3.10) is a more generalized extension and representation of the disturbance signal, therefore the parameter \(p\) is related to the complexity of the signal to estimate which provides extra information and increases the ability to track different types of disturbances. For example, \(p = 2\) allows local convergence to a disturbance with local constant derivative, \(p = 3\) allows local convergence to a disturbance with local constant acceleration, etc. Therefore, we can define the degree of approximation of the disturbance internal model by analyzing the complexity of the signal.

Notice that in order to obtain a good performance using a low order approximation (e.g. \(\frac{dT_r(t)}{dt} \approx 0\)) it would be necessary to design a high bandwidth observer. Of course, in those cases, the observer could give noisy estimations in practical applications. Then, higher degree of the internal model allow tuning the GPI observer with low-medium bandwidth.

### 3.3.3 Observer Design

**Theorem 3.1 (Aerodynamic Torque Observer)** Given Assumptions (1–4), the following GPI observer is proposed:

\[
\frac{d}{dt} \hat{x}(t) = A\hat{x}(t) + BT_g(t) + L^r (y(t) - C\hat{x}(t))
\]

(3.17)
where \( \hat{x}(t) = [\dot{x}_{wr}(t)^T \ \dot{x}_d(t)^T]^T \) is the augmented system state estimation vector and \( L^e_r = \begin{bmatrix} I^e_{p+3} & I^e_{p+2} & \cdots & I^e_2 & I^e_1 \end{bmatrix}^T \) is the observer gain vector. The GPI observer (3.17) asymptotically and exponentially reconstructs the system states \( \omega_r(t), \omega_g(t), \theta_s(t) \), and the disturbance inputs \( T_r(t), \dot{T}_r(t), \ldots, T_r^{(p-1)}(t) \) forcing the state estimation error \( \tilde{e}_x(t) = x(t) - \hat{x}(t) \) to converge towards the interior of a disk centered in the origin of the corresponding estimation error phase space, as long as the set of coefficients \( \{l^e_{p+3}, \ldots, l^e_2, l^e_1\} \) is chosen in such way that characteristic polynomial defined by

\[
\det(sI - A + L^e_r C) = 0
\]

is a Hurwitz polynomial.

**Proof.** See Appendix A.1. ■

**Remark 3.1** GPI observers are bandwidth limited by the roots location of the estimation error characteristic polynomial. Generally, the larger the observer bandwidth is, the more accurate the estimation will be. However, a large observer bandwidth will increase noise sensitivity. Then, the selection of the roots of the estimation error characteristic polynomial affects the bandwidth of the GPI observer and also the influence of measurement noises on the estimations. Therefore, GPI observers are usually tuned in a compromise between disturbance estimation performance (set by the internal model approximation degree) and noise sensitivity.

### 3.3.4 Results

The aerodynamic torque observer (3.17) proposed in Theorem 3.1 was implemented and tested on the nonlinear wind turbine benchmark model published by Odgaard et al. [42]. The parameters of the GPI observer were chosen as follows: \( p = 3, l^e_{p+3} = 1000, l^e_2 = 1028.22, l^e_3 = 1028.62, l^e_4 = 0.39, l^e_5 = 234.81 \) and \( l^e_6 = 28.46 \).

The estimate of the aerodynamic torque obtained using the GPI observer proposed in Theorem 3.1 is shown in Fig. 3.1. Then, from (3.9) and using \( \dot{T}_r(t) \), the estimation of the generator speed optimal reference trajectory \( \omega_{g,opt}(t) \) can be obtained as shown in Fig. 3.2.

### 3.4 GPI Control

Under the active disturbance rejection approach, a recent work is the control based on integral reconstructors which has been named Generalized Proportional Integral (GPI) control. This
Fig. 3.1 Aerodynamic torque estimation results on a 4.8MW wind turbine.

Fig. 3.2 Optimal angular speed trajectory calculation from aerodynamic torque.
control technique started in 2000 by M. Fliess, R. Márquez, E. Delaleau and H. Sira-Ramírez [67, 70] and involves in its design the active rejection of time-varying polynomial disturbances. This rejection is implemented using iterated integrals depending on the disturbance order to reject.

This section presents an alternative linear control technique based on robust GPI controllers to maximize wind energy capture in variable-speed wind turbines operating at partial load. The proposed strategy controls the tip-speed ratio $\lambda$ towards its optimal value by regulating the rotor angular speed to track an optimal reference trajectory. The GPI design technique solves the control problem through active rejection of all nonlinearities and disturbances of the WECS.

### 3.4.1 Control Design

From (1.8)-(1.10) and (1.12), we can derive the following representation of the rotor speed dynamics:

$$\omega_r(s) = \frac{J_g N_g^2 s^2 + (B_g N_g^2 + B_{dt} \eta_{dt}) s + K_{dt} \eta_{dt}}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} T_r(s) + \frac{-\alpha_{gc} N_g (B_{dt} s + K_{dt})}{(a_3 s^3 + a_2 s^2 + a_1 s + a_0) (s + \alpha_{gc})} T_{g,d}(s)$$

with,

$$a_3 = J_g J_r N_g^2$$
$$a_2 = (B_{dt} J_r \eta_{dt} + B_{dt} J_g N_g^2 + B_g J_r N_g^2 + B_r J_g N_g^2)$$
$$a_1 = (B_{dt} B_r \eta_{dt} + J_r K_{dt} \eta_{dt} + B_{dt} B_g N_g^2 + B_g B_r N_g^2 + J_g K_{dt} N_g^2)$$
$$a_0 = B_g K_{dt} N_g^2 + B_r K_{dt} \eta_{dt}$$

Now, reorganizing (3.19), consider the following simplified input-output system,

$$(s^4 + \gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0) \omega_r(s) = \kappa T_{g,d}(s) + \xi(s)$$

with,

$$\gamma_3 = \frac{(a_2 + \alpha_{gc} a_3)}{a_3}, \quad \gamma_2 = \frac{(a_1 + \alpha_{gc} a_2)}{a_3}, \quad \gamma_1 = \frac{(a_0 + \alpha_{gc} a_1)}{a_3}, \quad \gamma_0 = \frac{\alpha_{gc} a_0}{a_3},$$

$$\kappa = -\frac{1}{a_3} \alpha_{gc} N_g K_{dt},$$

$$\xi(s) = \frac{(s + \alpha_{gc})}{a_3} \left( J_g N_g^2 s^2 + (B_g N_g^2 + B_{dt} \eta_{dt}) s + K_{dt} \eta_{dt} \right) T_r(s).$$
where, \( \kappa, \gamma_0, \gamma_1, \gamma_2, \gamma_3 \) are known constants and \( \xi \) is a state dependent disturbance signal, that includes the effect of the external disturbance input \( T_r \). The unified disturbance signal \( \xi \) is assumed to be locally approximated to a time polynomial model (also called: Taylor polynomial) of order \( m \). Then, the disturbance input \( \xi \) is, hence, a time-varying function completely unknown but assumed to exhibit finitely uniformly, absolutely, bounded time derivatives,

\[
\sup_{t \geq 0} \left| \xi^{(m+1)}(t) \right| \leq K_\xi,
\]

which can be described by the following internal model:

\[
\frac{d^{m+1} \xi(t)}{dt^{m+1}} \approx 0. \tag{3.22}
\]

**Theorem 3.2 (ADR/GPI Control Law for Region 2) Assume an accurate estimation of \( T_r(t), \dot{T}_r(t), \ddot{T}_r(t), T_r^{(3)}(t), T_r^{(4)}(t) \) and \( \omega_r(t) \) provided by the Aerodynamic Torque Observer in Theorem 3.1. Then, for the system (3.21), the following control law

\[
T_{gd}(s) = \frac{1}{\kappa} \left[ T^*_g(s) - \frac{k_{m+4}s^{m+4} + \ldots + k_1s + k_0}{s^{m+1}(s^3 + k_{m+7}s^2 + k_{m+6}s + k_{m+5})} \left( \dot{\omega}_r(s) - \dot{\omega}_{r_{opt}}(s) \right) \right] \tag{3.23}
\]

with

\[
T^*_g(t) = \dot{\omega}^{(4)}_{r_{opt}}(t) + \gamma_3 \dot{\omega}^{(3)}_{r_{opt}}(t) + \gamma_2 \dot{\omega}_{r_{opt}}(t) + \gamma_1 \dot{\omega}_{r_{opt}}(t) + \gamma_0 \dot{\omega}_{r_{opt}}(t) \tag{3.24}
\]

\[
k = \frac{\lambda_{opt}}{\pi R^3 C_{opt}} \sqrt{\frac{2}{\rho R^2}} \tag{3.25}
\]

\[
\dot{\omega}_{r_{opt}}(t) = k \sqrt{\dot{T}_r(t)} \tag{3.26}
\]

\[
\ddot{\omega}_{r_{opt}}(t) = 0.5k \frac{\dot{T}_r(t)}{\dot{T}_r(t)} \tag{3.27}
\]

\[
\ddot{\omega}_{r_{opt}}(t) = \frac{0.5k \dot{T}_r(t)}{\dot{T}_r(t)^{0.5}} - \frac{0.25k \left( \dot{T}_r(t) \right)^{1.5}}{\dot{T}_r(t)^{0.5}} \tag{3.28}
\]

\[
\omega^{(3)}_{r_{opt}}(t) = \frac{0.5k \dot{T}_r(t)^{0.5}}{\dot{T}_r(t)^{0.5}} + \frac{0.375k \left( \dot{T}_r(t) \right)^{3}}{\dot{T}_r(t)^{2.5}} - \frac{0.75k \dot{T}_r(t) \dot{T}_r(t)}{\dot{T}_r(t)^{1.5}} \tag{3.29}
\]
\( \omega_{\text{opt}}^{(4)}(t) = \frac{0.5k\hat{T}_r(t)}{(\hat{T}_r(t))^{0.5}} - \frac{0.9375k(\hat{T}_r(t))^4}{(\hat{T}_r(t))^{3.5}} - \frac{0.75k(\hat{T}_r(t))^2}{(\hat{T}_r(t))^{1.5}} + \frac{2.25k(\hat{T}_r(t))^2\hat{T}_r(t)}{(\hat{T}_r(t))^{2.5}} - \frac{k\hat{T}_r(t)\hat{T}_r^{(3)}(t)}{(\hat{T}_r(t))^{1.5}} \) \hspace{1cm} (3.30)

asymptotically exponentially, uniformly, forces the closed loop tracking error \( e_{w_e}(t) = \omega_r(t) - \omega_{\text{opt}}(t) \) to converge towards the interior of a disk, centered around the origin in the tracking error space of phase coordinates, as long as the set of coefficients: \( \{k_{m+7}, ..., k_1, k_0\} \), are chosen in such a way that the polynomial, \( p_{w_e}(s) \), in the complex variable \( s \), defined by,

\[
p_{w_e}(s) = s^{m+1}(s^5 + k_{m+7}s^2 + k_{m+6}s + k_{m+5}) (s^4 + \gamma_3s^3 + \gamma_2s^2 + \gamma_1s + \gamma_0) + (k_{m+4}s^{m+4} + ... + k_1s + k_0)
\] \hspace{1cm} (3.31)

is a Hurwitz polynomial.

**Proof.** See Appendix A.2.

**Remark 3.2 (Internal model approximation of \( T_r(t) \) for Theorem 3.2)** In order to build the feedforward signal \( T_r(t) \) in (3.24) composed by (3.26)-(3.30), the internal model of the aerodynamic torque disturbance (see (3.10)) must be chosen as \( p \geq 5 \). If so, the aerodynamic torque observer (3.17) will be able to provide estimations of \( T_r(t), \hat{T}_r(t), T_r^{(3)}(t), T_r^{(4)}(t) \) and \( \omega_r(t) \), required to compose the control law (3.23).

**Remark 3.3 (Anti-windup implementation of the GPI controller)** From the control law (3.23), let us extract the controller and rewrite it as:

\[
U(s) = \frac{k_{m+4}s^{m+4} + ... + k_1s + k_0}{s^{m+1}(s^3 + k_{m+7}s^2 + k_{m+6}s + k_{m+5})} \hat{E}_{w_e}(s) = \frac{B(s)}{L(s)} \hat{E}_{w_e}(s)
\] \hspace{1cm} (3.32)

where \( \hat{E}_{w_e}(s) = \hat{\omega}_r(s) - \hat{\omega}_{\text{opt}}(s) \), \( L(s) = s^{m+4} + k_{m+7}s^{m+3} + k_{m+6}s^{m+2} + k_{m+5}s^{m+1} \) and \( B(s) = k_{m+4}s^{m+4} + ... + k_1s + k_0 \). Then, the controller (3.32) can be implemented by means of the saturation feedback as shown in Fig. 3.3, where the polynomial \( E_1(s) \) can be, in principle, any monic stable polynomial of order \( m+4 \).

The performance of this anti-windup scheme depends on the appropriate choice of the polynomial \( E_1(s) \). From Fig. 3.3, notice that when there is no saturation, the controller can be expressed as \( U(s) = \frac{B(s)}{E_1(s)} \hat{E}_{w_e}(s) - \frac{L(s)}{E_1(s)} E_1(s) U(s) \) and then \( U(s) = \frac{B(s)}{L(s)} \hat{E}_{w_e}(s) \), which is the same controller’s dynamics given in (3.32). On the other hand, when there is saturation, the dynamics of the polynomial \( E_1(s) \) acts in benefit of the controller’s behavior.
Fig. 3.3 Anti-windup implementation of the GPI controller.

3.4.2 Results

Fig. 3.4 Closed-loop system scheme of the proposed ADR/GPI control strategy.

The proposed GPI aerodynamic torque observer (see Theorem 3.1) and the ADR/GPI control law (see Theorem 3.2) were implemented and tested on the nonlinear wind turbine benchmark model [42]. Fig. 3.4 shows the block diagram of the proposed control strategy. The unified disturbance function $\hat{\xi}$ was chosen as a first-order Taylor approximation, i.e. $(m = 1)$, then the robust GPI control law is:

$$T_{g,d}(s) = \frac{1}{\kappa} \left[ T^*_{g}(s) - \frac{k^c_5 s^5 + k^c_4 s^4 + k^c_3 s^3 + k^c_2 s^2 + k^c_1 s + k^c_0}{s^2 (s^3 + k^c_8 s^2 + k^c_7 s + k^c_6)} (\hat{\omega}_r(s) - \hat{\omega}_{opt}(s)) \right]. \quad (3.33)$$

Replacing (3.33) in (3.21), the tracking error dynamics is given by

$$p_{\omega_r}(s) E_{\omega_r}(s) = \hat{\xi}(s) s^2 \left( s^3 + k^c_8 s^2 + k^c_7 s + k^c_6 \right). \quad (3.34)$$
The parameters of the control law were chosen as follows: \( m = 1, k_c^1 = 1.0493, k_c^2 = 310459.61, k_c^3 = 49348815.26, k_c^4 = 10991.73, k_c^5 = 1212466.54, k_c^6 = 192500, k_c^7 = 25003.98, k_c^8 = 5764.18 \) and \( k_c^9 = 5.058 \).

The results of the aerodynamic torque estimation and optimal setpoint generation are shown and compared in Fig. 3.5. By examining the optimal generator speed profile, it can be seen that in order to achieve a compromise between energy capture maximization and dynamic loads reduction, an intermediate observing dynamics should be chosen in the aerodynamic torque observer (3.17). Notice that \( \hat{\omega}_{g_{opt}}(t) \) only tracks the mean tendency of the optimal generator speed \( \omega_{g_{opt}}(t) \), while avoiding the tracking of the short-time turbulence.

The Fig. 3.6 shows the simulation results of the entire proposed control strategy. The control system tracks the mean tendency of the optimal reference trajectory to force the WECS power coefficient \( C_P \) close to its optimal value (see Power Coefficient and Generator angular speed in Fig. 3.6). The tracking error of the control system is around zero as shown in Fig. 3.6. A medium performance on the control gains \( \{k_c^0, k_c^1, ..., k_c^8\} \) was selected in order to avoid tracking fast changes of the optimal generator speed. Therefore, the rest of the fast fluctuations of the aerodynamic torque transferred to the generator speed optimal trajectory \( \hat{\omega}_{g_{opt}}(t) \) are not tracked (see Generator angular speed in Fig. 3.6).

The Fig. 3.6 details the aerodynamic power captured by the proposed ADR/GPI control law and the evolution in time of the WECS power coefficient. It is observed that the captured
Fig. 3.6 Simulation results of the proposed ADR/GPI robust control law.

The aerodynamic power with the proposed control law is greater than the power captured by the standard torque control. In addition, it is noticed that the proposed control strategy forces the power coefficient to be closer to its optimal value $C_{P_{opt}} = 0.4554$ than the standard torque control, which allows better power capture.

The performance of each control system is compared using an aerodynamic efficiency index $\eta_{aero}$ [86], which is defined as follows:

$$\eta_{aero} = \frac{\int_{t_i}^{t_f} P_r(t)dt}{\int_{t_i}^{t_f} P_{r_{opt}}(t)dt}$$

The evaluation of the criteria defined in (3.35) stated that the aerodynamic efficiency obtained by the proposed ADR/GPI control approach is 98.84%, while the efficiency of the baseline controller is 95.77%.

The benchmark model [42] contains faults which require the control system to be re-configured to continue power generation, as well as very severe faults which require a safe
ADRC approach to Maximize Energy Capture in Wind Turbines

Fig. 3.7 Simulation results of the ADR/GPI Control law on power converter fault.

and fast shut down of the wind turbine. In order to evaluate the capability of handling such system changes, a typical malfunction in the internal power converter control loops is used. As a consequence, this non-severe fault must be accommodated in some way and the wind turbine must continue its operation. The fault considered is an offset, denoted as $\delta T_g$, on the generator torque, which can be caused by an error in the initialization of the converter controller [138]. The converter offset is configured to $\delta T_g = 5000$ N-m.

The Fig. 3.7 shows the closed-loop performance of both the standard torque control and the proposed ADR/GPI control approach under the actuator fault. The fault occurs from 200s to 400s as seen in Fig. 3.7. It is observed that the disturbance is rejected by the proposed ADR/GPI control approach and the aerodynamic power captured maintains approximately with no changes. The figure also shows that the power coefficient of the WECS is still close to its optimal value. On the other hand, the standard torque control of the benchmark cannot handle the actuator fault and much of the aerodynamic power is lost. The evaluation of the criteria defined in (3.35) stated that the aerodynamic efficiency obtained using the proposed ADR/GPI control approach is $98.79\%$, while the efficiency of the standard torque controller is $85.88\%$. 
Fig. 3.8 Closed-loop frequency response using the proposed ADR/GPI control law.

Fig. 3.9 Open-loop frequency response using the proposed ADR/GPI control law.
3.4.3 Robustness

For the second case, robustness of the control system is verified through Nyquist plots of the open-loop control system by measuring the modulus margin $\Delta_M$. This analysis is carried out for $\pm 25\%$ of uncertainty in each parameter of the WECS. Fig. 3.9 details the Nyquist plots of the proposed control system in open-loop and reveals that the modulus margin in the nominal case (green) is $\Delta_M = 0.655$ and for the worst case (red) the modulus margin is $\Delta_M = 0.425$; this demonstrates robust stability. In addition, closed-loop performance of the control system can be verified in the Bode diagram of Fig. 3.8. These plots show the nominal frequency response, the frequency response with $\pm 25\%$ of uncertainty in each parameter of the WECS, and the frequency response in the worst case.

3.5 GPI Observer-Based Control

GPI observer-based control of linear and nonlinear uncertain systems is very much related to methodologies known as disturbance accommodation control (DAC) [74] and active disturbance rejection control (ADRC) [75–77]. These approaches deal with the problem of cancelling, from the controller’s actions, endogenous and exogenous unknown additive disturbance inputs affecting the system. Equivalent input disturbances are made available via suitable linear or nonlinear estimation.

This section presents an alternative linear control technique based on GPI observers to maximize wind energy capture in variable-speed wind turbines operating at partial load. The proposed strategy uses a GPI observer to reconstruct the aerodynamic torque in order to provide a generator speed optimal trajectory to a robust GPI observer-based controller that regulates the power coefficient, via the generator torque, towards an optimum point at which the power coefficient is maximum. The proposed GPI observer-based control technique adds robustness to the system and solves the control problem through linear active estimation and rejection of nonlinearities and disturbances of the wind energy conversion system.

3.5.1 Control Design

Based on (1.9), the generator angular speed satisfies the following dynamics:

$$\frac{d}{dt}\omega_g(t) = -\frac{1}{J_g} T_g(t) + \frac{K_{dt}}{J_g N_g} \theta_s(t) + \frac{B_{dt}}{J_g N_g} \omega_r(t) - \left( \frac{B_{ls}}{J_g N_g^2} + \frac{B_{ds}}{J_r} \right) \omega_g(t)$$

(3.36)
Then, reorganizing and lumping together some terms of (3.36), the following simplified system is obtained:

\[ \dot{\omega}_g(t) = \kappa T_g(t) + \varphi(t) + \Delta_1(t) \]  

(3.37)

with

\[ \kappa = -\frac{1}{J_g} \]

\[ \varphi(t) = \frac{K_{dt}}{J_g N_g} \theta_s(t) + \frac{B_{dt}}{J_g N_g} \omega_r(t) - \left( \frac{B_{dt}}{J_g N_g^2} + \frac{B_{hs}}{J_g} \right) \omega_g(t) \]

where \( \kappa \) is a known constant, \( \varphi(t) \) is a state dependent input disturbance, and \( \Delta_1(t) \) is an input disturbance function that lumps together all the uncertainty associated to the system. The disturbance \( \Delta_1(t) \) contains the rest of the system dynamics (actuator), including some unmodeled dynamics, disturbances of additive nature, actuator faults, parameter variations, and nonlinear effects of the WECS.

In relation to the simplified system (3.37), the following assumptions are stated:

1. For a sufficiently large positive integer \( m \), the disturbance input \( \Delta_1(t) \) exhibits uniformly absolute bounded time derivative of order \( m \). This condition assures the existence of an unknown but finite constant, \( K_{\Delta_1} \), such that

\[ \sup_{t \geq 0} \left| \Delta_1^{(m)}(t) \right| \leq K_{\Delta_1}. \]

2. The unknown input disturbance \( \Delta_1(t) \) can be modeled by the approximation of its internal model given by

\[ \frac{d^m \Delta_1(t)}{dt^m} \approx 0 \]  

(3.38)

Consider the following disturbance states, related to (3.38),

\[ x_{\Delta_1}(t) = \begin{bmatrix} \Delta_1(t) & \dot{\Delta}_1(t) & \cdots & \Delta_1^{(m-2)}(t) & \Delta_1^{(m-1)}(t) \end{bmatrix}^T \]  

(3.39)

where their corresponding dynamics is given by:

\[ \frac{d}{dt} x_{\Delta_1}(t) = A_{\Delta_1} x_{\Delta_1}(t) + B_{\Delta_1} \Delta_1^{(m)}(t) \]

\[ \Delta_1(t) = C_{\Delta_1} x_{\Delta_1}(t) \]  

(3.40)
ADRC approach to Maximize Energy Capture in Wind Turbines

with

\[
A_{\Delta_1} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix},
B_{\Delta_1} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix},
C_{\Delta_1} = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0
\end{bmatrix}
\]

where \( x_{\Delta_1}(t) \in \mathbb{R}^{m \times 1}, A_{\Delta_1} \in \mathbb{R}^{m \times m}, B_{\Delta_1} \in \mathbb{R}^{m \times 1} \) and \( C_{\Delta_1} \in \mathbb{R}^{1 \times m} \).

Then, it is possible to augment the simplified system (3.37) with the unknown input disturbance state vector \( x_{\Delta_1}(t) \); thus,

\[
\frac{d}{dt} x_c(t) = A_c x_c(t) + B_{c_1} T_g(t) + B_{c_2} \phi(t) + B_{c_3} \Delta_1^{(m)}(t)
\]

\[
y(t) = C_c x_c(t)
\]

with

\[
x_c(t) = \begin{bmatrix}
\omega_g(t) \\
x_{\Delta_1}(t)
\end{bmatrix},
A_c = \begin{bmatrix}
0 & C_{\Delta_1} \\
0 & A_{\Delta_1}
\end{bmatrix},
B_{c_1} = \begin{bmatrix}
\kappa \\
0
\end{bmatrix},
B_{c_2} = \begin{bmatrix}
1 \\
0
\end{bmatrix},
B_{c_3} = \begin{bmatrix}
0 \\
B_{\Delta_1}
\end{bmatrix},
C_c = \begin{bmatrix}
1 & 0
\end{bmatrix}
\]

where \( x_c(t) \in \mathbb{R}^{(m+1) \times 1}, A_c \in \mathbb{R}^{(m+1) \times (m+1)}, B_{c_1}, B_{c_2}, B_{c_3} \in \mathbb{R}^{(m+1) \times 1} \) and \( C_c \in \mathbb{R}^{1 \times (m+1)} \).

It is desired that the generator angular speed \( \omega_g(t) \) accurately tracks the optimal reference trajectory \( \omega_g^{opt}(t) \), with tracking error defined by \( e_y(t) = \omega_g(t) - \omega_g^{opt}(t) \) absolutely bounded by a small quantity \( \epsilon \); that is, \( \sup_{t \geq 0} |e_y(t)| \leq \epsilon \). Then, based on (3.37), (3.38), and (3.41), the following GPI observer-based control is proposed.

**Theorem 3.3 (Disturbance \( \Delta_1(t) \) observer)** Given Assumptions 5 and 6, the estimation of the disturbance function \( \Delta_1(t) \), denoted as \( \hat{\Delta}_1(t) \), is given by the following GPI observer:

\[
\frac{d}{dt} \hat{x}_c(t) = A_c \hat{x}_c(t) + B_{c_1} T_g(t) + B_{c_2} \hat{\phi}(t) + L_{\Delta_1}(y(t) - C_c \hat{x}_c(t))
\]

\[
\hat{\Delta}_1(t) = \begin{bmatrix}
0 & C_{\Delta_1}
\end{bmatrix} \hat{x}_c(t)
\]

with

\[
\hat{\phi}(t) = \frac{K_{dt}}{J_g N_g^2} \hat{T}_s(t) + \frac{B_{dt}}{J_g N_g} \hat{\phi}_s(t) - \left( \frac{B_{dt}}{J_g N_g^2} + \frac{B_{hs}}{J_g} \right) \hat{\omega}_g(t)
\]
where \( \hat{x}_c(t) = \begin{bmatrix} \hat{\omega}_g(t) & \hat{\Delta}_1(t) & \hat{\Delta}_1(t) & \cdots & \hat{\Delta}_1^{(m-2)}(t) & \hat{\Delta}_1^{(m-1)}(t) \end{bmatrix}^T \) is the estimated system state vector, \( L_{\Delta_1} = \begin{bmatrix} l_{m+1}^{\Delta_1} & l_{m}^{\Delta_1} & \cdots & l_{2}^{\Delta_1} & l_{1}^{\Delta_1} \end{bmatrix}^T \) is the observer gain vector, and \( \hat{\phi}(t) \) is the estimation of \( \phi(t) \) reconstructed by using the states of the aerodynamic torque observer given in (3.17). The observer (3.42) asymptotically and exponentially reconstructs the disturbance \( \Delta_1(t) \), forcing the state estimation error \( \tilde{e}_x(t) = x_c(t) - \hat{x}_c(t) \) to converge towards the interior of a disk centered in the origin of the corresponding estimation error phase space, provided the set of coefficients \( \{ l_{m+1}^{\Delta_1}, \ldots, l_{2}^{\Delta_1}, l_{1}^{\Delta_1} \} \), which are chosen in such way that the polynomial \( P_{\Delta_1}(s) \), in the complex variable \( s \), defined by

\[
P_{\Delta_1}(s) = s^{m+1} + l_{m+1}^{\Delta_1}s^m + \cdots + l_{2}^{\Delta_1}s + l_{1}^{\Delta_1}
\]

(3.44)
is a Hurwitz polynomial, with roots located to the left of the imaginary axis of the complex plane.

**Proof.** See Appendix A.3.  ■

**Theorem 3.4 (ADR Observer-based Control Law for Region 2)** Assume an accurate estimation of \( \phi(t), \Delta_1(t), T_c(t) \) and \( \dot{T}_c(t) \); then, for the simplified system (3.37), the following control law is proposed:

\[
T_g(t) = \frac{1}{\kappa} \left[ \hat{\omega}_{g_{opt}}(t) - k_0^c \left( \omega_g(t) - \hat{\omega}_{g_{opt}}(t) \right) - \hat{\phi}(t) - \hat{\Delta}_1(t) \right]
\]

(3.45)

with

\[
\hat{\omega}_{g_{opt}}(t) = \frac{N_g \lambda_{opt}}{R} \sqrt{\frac{2\lambda_{opt}}{\rho \pi R^3 C_{p_{opt}}}} \hat{T}_c(t)
\]

(3.46)

\[
\hat{\omega}_{g_{opt}}(t) = \frac{N_g \lambda_{opt}}{2R \sqrt{\hat{T}_c(t)}} \sqrt{\frac{2\lambda_{opt}}{\rho \pi R^3 C_{p_{opt}}}} \dot{\hat{T}_c}(t)
\]

(3.47)

where \( \hat{T}_c(t) \) and \( \hat{\dot{T}_c}(t) \) are provided by the aerodynamic torque observer given in Theorem 3.1, \( \hat{\phi}(t) \) is reconstructed by using the states of the aerodynamic torque observer given in (3.17), and \( \hat{\Delta}_1(t) \) is provided by the GPI disturbance observer given in Theorem 3.3. Such control law asymptotically and exponentially forces the closed loop system tracking error \( e_y(t) = \omega_g(t) - \omega_{g_{opt}}(t) \) to converge towards the interior of a disk of radius as small as desired centered in zero, provided that the coefficient is \( k_0^c > 0 \).

**Proof.** See Appendix A.4.  ■
Remark 3.4 (Internal model approximation of $T_r(t)$ for Theorem 3.4) In order to build the signals $\hat{\omega}_{\text{gopt}}(t)$ and $\hat{\dot{\omega}}_{\text{gopt}}(t)$ in (3.46) and (3.47), respectively; the internal model of the aerodynamic torque disturbance (see (3.10)) must be chosen as $p \geq 2$. If so, the aerodynamic torque observer (3.17) will be able to provide estimations of $T_r(t)$ and $\dot{T}_r(t)$ required to compose the control law (3.45).

Remark 3.5 Note that for the GPI observer-based control strategy defined in (3.42) and (3.45), the energy capture maximization depends on the accurate reconstruction of the optimal reference trajectory $\omega_{\text{gopt}}(t)$ and $\dot{\omega}_{\text{gopt}}(t)$.

3.5.2 Zero dynamics

Since the relative order of the WECS is one and system order is three, zero dynamics comes into play and in consequence must be analyzed. Considering the third-order system dynamics defined in (1.8)-(1.10), the zero dynamics is given by $\omega_r(t)$ and $\dot{\theta}_s(t)$ with $\omega_g(t)$ set to zero:

$$
\begin{bmatrix}
\dot{\omega}_r(t) \\
\dot{\theta}_s(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{-B_{dt} + B_{ls}}{J_r} & -K_{dt} \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\omega_r(t) \\
\dot{\theta}_s(t)
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{J_r} \\
0
\end{bmatrix} T_r(t)
$$

$$
|sI - A_z| = s^2 + \left(\frac{B_{dt} + B_{ls}}{J_r}\right) s + \frac{K_{dt}}{J_r}
$$

Then, the internal dynamics is now given by the eigenvalues of $A_z$, which are all stable since the parameters of the WECS $B_{dt}$, $B_{ls}$, $K_{dt}$ and $J_r$ are positive. However, some troubles may arise in the control system when the internal dynamics is poorly damped. In such cases (when the internal dynamics became problematic) some methods can be used [45] to add damping to the drive-train system.

3.5.3 Results

The proposed aerodynamic torque GPI-observer (see Theorem 3.1) and the GPI observer-based control (see Theorems 3.3 and 3.4) were implemented and tested on the nonlinear wind turbine benchmark model [42]. Fig. 3.10 shows the block diagram of the proposed control strategy. The parameters of the observers and the control strategy were chosen as follows: $m = 3$, $k_0^c = 1$, $l_1^{\Delta_1} = 39.38$, $l_2^{\Delta_1} = 61.69$, $l_3^{\Delta_1} = 35.30$ and $l_4^{\Delta_1} = 8.66$.

The simulation results of the proposed control strategy are shown in Fig. 3.11. Under nominal conditions, the control system tracks the optimal reference trajectory to force the WECS power coefficient $C_P$ close to its optimal value (see Power Coefficient plot in Fig.
3.5 GPI Observer-Based Control

Despite of the external disturbances and nonlinearities of the benchmark model, the tracking error of the control system is near to zero as shown in Fig. 3.11. In order to achieve a good compromise between energy capture and dynamic loads on the low speed shaft, a medium performance on the control gain $k_c^0$ was selected. Therefore, most of the fast fluctuations of the aerodynamic torque transferred to the generator speed optimal trajectory $\hat{\omega}_{g,\text{opt}}(t)$ are not tracked (see Generator angular speed in Fig. 3.11).

Fig. 3.11 details the aerodynamic power captured by the GPI observer-based control and the evolution in time of the WECS power coefficient, respectively. It is observed that the captured aerodynamic power with the proposed control is greater than the power captured by the standard torque control. In addition, it is noticed that the proposed control strategy forces the power coefficient close to its optimal value $C_{P,\text{opt}} = 0.4554$, which allows better power capture.

The performance of each control system is compared using an aerodynamic efficiency index $\eta_{\text{aero}}$ [86]. The evaluation of the criteria defined in (3.35) stated that the aerodynamic efficiency obtained using the proposed ADR/GPI Observer-based control approach is 98.8%, while the efficiency of the baseline controller is 95.77%.

The benchmark model [42] contains faults which require the control system to be reconfigured to continue power generation, as well as very severe faults which require a safe and fast shut down of the wind turbine. Here, in order to evaluate the active disturbance rejection capability of the proposed GPI observer-based control strategy, a typical malfunction in the internal power converter control loops is used. As a consequence, this non-severe fault must be accommodated in some way and the wind turbine must continue its operation. The fault
ADRC approach to Maximize Energy Capture in Wind Turbines

considered is an offset, denoted as $\delta T_g$, on the generator torque, which can be caused by an error in the initialization of the converter controller [138]. The converter offset is configured to $\delta T_g = 5000 \text{ N-m}$.

Fig. 3.12 shows the closed loop performance of both the standard torque control and the proposed GPI observer-based control approach under the actuator fault. The fault occurs from 200s to 400s as seen in Fig. 3.12. When the fault becomes active (Fault state = 0), the GPI-observer in Theorem 3.3, via the observer state $\hat{\Delta}_1(t)$, estimates the disturbance function on-line and actively rejects it by canceling its effect through the control law (3.45).

It is observed in Fig. 3.12 that the disturbance is rejected by the proposed ADR observer-based control approach and almost any lost in the aerodynamic power captured can not be easily appreciated. The Fig. also shows that the power coefficient of the WECS is still close to its optimal value. On the other hand, the standard torque control of the benchmark cannot handle the actuator fault and much of the aerodynamic power is lost. The evaluation of the criteria defined in (3.35) stated that the aerodynamic efficiency obtained by the proposed

Fig. 3.11 Simulation results of the proposed GPI observer-based control.
Fig. 3.12 Simulation results using the proposed ADR/GPI Observer-based control approach on power converter fault.
ADR/GPI observer-based control approach is 98.65%, while the efficiency of the standard torque controller is 85.88%.

The Fig. 3.13 shows power capture results of the proposed control approach when the ADR/GPI observer in Theorem 3.3 is not used. The plots show that when there is no actuator fault, the proposed control approach with or without the ADR/GPI observer performs with no change. However, the Fig. 3.13 also shows that the ADR/GPI observer is useful when an unknown disturbance is affecting the system, resulting in better power capture. The aerodynamic efficiency obtained using the proposed ADR/GPI observer-based control approach is 98.65%, against 97.85% when the ADR/GPI observer in Theorem 3.3 is not used.
3.6 Conclusions

3.6.1 ADR/GPI Control

A control strategy based on robust GPI controllers has been proposed for disturbance tracking of variable-speed wind turbines operating at partial load. The control law was proposed to track an optimal trajectory of the rotor speed towards an optimum point of the aerodynamic coefficient $C_{P_{opt}}$ of the WECS. The proposed design strategy solved the control problem in a simple way but also providing robustness against actuator changes and external disturbances.

In order to create the optimal trajectory of the WECS for partial load operation, a GPI observer was proposed to accurately estimate the aerodynamic torque $T_r(t)$ and a set of its derivatives ($\dot{T}_r(t), \ddot{T}_r(t), T_r^{(3)}(t)$ and $T_r^{(4)}(t)$). Then, based on those estimations, the optimal rotor speed trajectories ($\hat{\omega}_r^{opt}(t), \hat{\omega}_r^{opt}(t), \hat{\omega}_r^{opt}(t), T_r^{(3)}(t)$ and $T_r^{(4)}(t)$) can be composed and injected into the ADR/GPI control law, avoiding the need of wind speed estimation or measurement.

Several simulation tests were performed on a nonlinear benchmark model using the proposed robust GPI control law. The results showed that the captured wind energy was maximized even when an actuator fault was applied.

3.6.2 ADR/GPI Observer-based Control

A linear active disturbance rejection control strategy based on two GPI observers for maximum wind energy capture of variable-speed wind turbines operating at partial load has been proposed. In order to create the generator speed optimal trajectory towards an optimum point at which the WECS power coefficient is maximum, an ADR philosophy-based GPI observer was formulated to estimate the aerodynamic torque $T_r(t)$ and its first derivative $T_r(t)$. Then, an ADR philosophy-based GPI observer-based controller was proposed to absolutely and arbitrarily bound the generator speed tracking error.

The proposed strategy solved the control problem based on linear active estimation of possible nonlinearities and disturbances of the WECS, and these accurate estimations were used by a simplified linear control law, in which the captured wind energy was maximized.

It was shown through simulation tests on a nonlinear benchmark model that the proposed dual GPI observer control strategy maximized the captured wind energy even when an actuator fault was applied. This demonstrates the robustness added by the GPI observer-based control.

It is worth noting that the proposed control strategy is related to exact feedback linearization, but there are some important differences between both strategies which give advantages
to ADR/GPI observer-based control, such as the following: (a) GPI observer-based control does not require system state measurements, (b) any mismatch between the system model and the real system is lumped together in a disturbance function $\Delta_1(t)$ that is estimated and rejected on-line, (c) GPI observers are capable of estimating a certain number of disturbance function derivatives (useful to determine $\dot{\omega}_{\text{opt}}$), and (d) ADR philosophy plays a very important role in GPI observer-based control since the internal model of the disturbance function is taken into account in the design process.
Chapter 4

ADRC approach of Wind Turbines Operating in Full-Load Region

4.1 Introduction

The technology developed to get benefit from wind energy has risen from the experimental to be nowadays the highest growth rate renewable energy source in the world [1]. In fact, among the current exploited renewable sources, Wind Energy Conversion Systems (WECS) are considered as the most cost effective approach [60]. The larger the wind turbines size is the more wind energy can be captured. Therefore, big turbines have economic advantages and this is evidenced by the impressive sizes of the wind turbines today (like a Boeing 747 or a football field [4]).

Large modern wind turbines are machines with enormous challenges when operating in full-load region, not only because of the regulation of speed and power under highly nonlinear aerodynamics, but also due to the high efficiency required even when model uncertainties, external disturbances, or system faults are present. As a consequence, the efficiency of power capture and power generation is strongly dependent on the selected control method [50] and this represents an important potential of research and development in science and engineering. This situation provides a motivation to consider new alternative control techniques to make the WECS more efficient and reliable.

The need for robust pitch controllers that provide an accepted performance and disturbance rejection in the full-load region of operation of the wind turbine has been evident, and several studies have been carried out to solve the problem. Gain scheduling control of wind turbines [44, 105, 139] have been studied and robust linear parameter-varying (LPV) controllers based on LPV models [45, 116] have been proposed. Also, integral sliding mode
controllers [54], robust LTI $H_\infty$ controllers [60] and a frequency domain gain scheduling control [59] have been proposed. However, those control schemes may result in very complex control systems and some of them do not use the knowledge of disturbances in order to reject them all or at least a part of them.

Some of the active-disturbance-rejection- (ADR-) based techniques allow linear control solutions for some class of uncertain complex nonlinear systems. The ADR philosophy, is a control paradigm started by Prof. G. Shipanov and Prof. Jingqing Han [75, 140]. Also, under the same approach, Generalized Proportional Integral (GPI) control [67, 70] is very related to Active Disturbance Rejection Control (ADRC). Nowadays, ADRC and GPI control have been extended and applied by Prof. Z. Gao, H. Sira-Ramírez and others (see [49, 68, 69, 76, 77, 141–144]). In ADRC and GPI control, the disturbances, unmodeled dynamics and parameter uncertainty are treated as a lumped disturbance signal. Then, this unified disturbance signal is estimated on-line with a pre-defined level of approximation (knowledge) and then used in the control law to approximately cancel it. Therefore, ADR schemes could offer a linear, simpler, and robust solution for controlling wind turbines operating in full-load region.

4.2 Wind turbine model

Here, a derivation of a linear uncertain model based on the non-linear horizontal-axis wind turbine described in section 1.4 is presented [21, 44–46]. Refer to section 1.4 to detail the aerodynamic, structural, pitch and generator models. For this study the 5 MW reference wind turbine implemented in the FAST code [78] is used. FAST is considered as a standard wind turbine dynamic simulation tool in industry and will be used in this chapter to validate the open-loop and the closed-loop results. Nominal numerical values of each parameter of the wind turbine are listed in Table 4.1 [46, 78].

4.2.1 Operating trajectory of the wind turbine

From operating regions 2 and 3, a curve for the optimal operating trajectory of a wind turbine can be obtained. This curve corresponds to the steady-state values of collective pitch angle and TSR due to constant wind speeds evaluated along the entire span of the wind turbine. The Fig. 4.1 shows the optimal operating trajectory of a 5MW reference wind turbine implemented in the FAST code [78] with $\bar{\lambda} = \frac{R\phi_c}{V_w}$. The operating trajectory in region 3 of the 5MW reference wind turbine is used to propose a robust ADRC scheme for wind speeds between 11.4m/s and 25m/s.
Table 4.1 Parameters of the 5 MW wind turbine used for the study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$12445.26 \text{ m}^2$</td>
<td>$P_N$</td>
<td>$5296.6101694 \text{ kW}$</td>
</tr>
<tr>
<td>R</td>
<td>$63 \text{ m}$</td>
<td>$\omega_N$</td>
<td>$12.1 \text{ rpm}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$1.225 \text{ kg/m}^3$</td>
<td>$B_{ls}$</td>
<td>$1000 \text{ Nm/(rad/s)}$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>97</td>
<td>$B_{hs}$</td>
<td>$1 \text{ Nm/(rad/s)}$</td>
</tr>
<tr>
<td>$r_b$</td>
<td>$30.64 \text{ m}$</td>
<td>$\tau$</td>
<td>$0.1 \text{ s}$</td>
</tr>
<tr>
<td>$J_g$</td>
<td>$534.116 \text{ kgm}^2$</td>
<td>$J_r$</td>
<td>$38759227 \text{ kgm}^2$</td>
</tr>
<tr>
<td>$m_p$</td>
<td>$12024 \text{ kg}$</td>
<td>$m_t$</td>
<td>$656503.178 \text{ kg}$</td>
</tr>
<tr>
<td>$\omega_{lb}$</td>
<td>$4.394 \text{ rad/s}$</td>
<td>$\omega_{lt}$</td>
<td>$2.036 \text{ rad/s}$</td>
</tr>
<tr>
<td>$K_b$</td>
<td>$232119 \text{ N/m}$</td>
<td>$B_b$</td>
<td>$504.1 \text{ Ns/m}$</td>
</tr>
<tr>
<td>$K_t$</td>
<td>$2721400 \text{ N/m}$</td>
<td>$B_t$</td>
<td>$26732.81 \text{ Ns/m}$</td>
</tr>
<tr>
<td>$K_{dt}$</td>
<td>$867637000 \text{ Nm/rad}$</td>
<td>$B_{dt}$</td>
<td>$6210000 \text{ Nm/(rad/s)}$</td>
</tr>
</tbody>
</table>

Fig. 4.1 Optimal operating trajectory of the wind turbine ($\bar{\omega_r}, \bar{\beta}, \bar{V_w}$).

4.2.2 Open-loop uncertain system model

A Jacobian linearization method is applied to obtain an uncertain model of the nonlinear wind turbine model. Then, the nonlinear aerodynamic torque $T_r(t)$ (1.2) and the thrust force $F_T(t)$ (1.3) are linearized around the operating trajectory $\Theta = (\bar{\omega_r}, \bar{\beta}, \bar{V_w})$\(^1\) as follows:

\[
\Delta T_r(t) = -B_r\omega(\Theta)\Delta \omega_r(t) + k_r(\Theta)\Delta \beta(t) + k_rv(\Theta)\Delta V_e(t) \\
\Delta F_T(t) = -B_T(\Theta)\Delta \omega_r(t) + k_T(\Theta)\Delta \beta(t) + k_Tv(\Theta)\Delta V_e(t)
\] (4.1)

\(^1\)The bar sign over the variables denotes their mean value or equilibrium point at each operating point.
where,

\[ B_{r\omega}(\Theta) = -\frac{\partial T_r}{\partial \omega} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]
\[ B_T(\Theta) = -\frac{\partial F_T}{\partial \omega} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]
\[ k_{r\beta}(\Theta) = \frac{\partial T_r}{\partial \beta} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]
\[ k_T\beta(\Theta) = \frac{\partial F_T}{\partial \beta} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]
\[ k_{rv}(\Theta) = \frac{\partial T_r}{\partial V_e} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]
\[ k_Tv(\Theta) = \frac{\partial F_T}{\partial V_e} (\bar{\omega}, \bar{\beta}, \bar{V}_w) \]

(4.2)

with \( \Delta \omega_r(t) = \omega_r(t) - \bar{\omega}_r \), \( \Delta \beta(t) = \beta(t) - \bar{\beta} \), \( \Delta T_r(t) = T_r(t) - \bar{T}_r \), and \( \Delta F_T(t) = F_T(t) - \bar{F}_T \).

The Fig. 4.2 shows the variation of each parameter of (4.2), evaluated along the optimal operating trajectory of the wind turbine and the Table 4.2 shows the maximum and minimum values of each uncertain parameter in (4.2).

Table 4.2 Maximum and minimum values for the linear parameter varying terms of the uncertain model of the 5 MW wind turbine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{r\omega}(\Theta) )</td>
<td>3486898.024</td>
<td>22306846.662</td>
</tr>
<tr>
<td>( B_T(\Theta) )</td>
<td>26626.351</td>
<td>1111962.786</td>
</tr>
<tr>
<td>( k_{r\beta}(\Theta) )</td>
<td>-1402905.782</td>
<td>-431498.171</td>
</tr>
<tr>
<td>( k_T\beta(\Theta) )</td>
<td>-78159.726</td>
<td>-61301.338</td>
</tr>
<tr>
<td>( k_{rv}(\Theta) )</td>
<td>985094.802</td>
<td>1470344.223</td>
</tr>
<tr>
<td>( k_Tv(\Theta) )</td>
<td>75998.862</td>
<td>78758.125</td>
</tr>
</tbody>
</table>

Fig. 4.2 Partial derivatives of \( F_T \) and \( T_r \) evaluated along the optimal operating trajectory.
\begin{align*}
A_t(\Theta) &= \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-\frac{(m_t+Nm_b)K_b}{m_b} & -\frac{K_t}{m_t} & -\frac{k_T(\Theta)}{m_b} & -\frac{(m_t+Nm_b)B_b}{m_b} & \frac{B_t}{r_pm_t} & -\frac{k_T(\Theta)}{m_b r_b} & 0 & 0 \\
-\frac{Nr_b K_b}{m_t} & -\frac{K_t}{m_t} & \frac{K_{r_b}}{J_r} & -\frac{k_T(\Theta)}{m_b} & -\frac{B_t}{r_pm_t} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{K_{d_t}}{J_{r_c} N_g} & -\frac{B_{d_t}}{J_{r_c} N_g} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \\
B_{ut} &= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\frac{k_T(\Theta)}{m_b r_b} \\
\frac{k_T(\Theta)}{J_r} \\
\frac{k_T(\Theta)}{J_r} \\
\frac{1}{\tau}
\end{bmatrix},
B_{wt} &= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\frac{k_T(\Theta)}{m_b r_b} \\
\frac{k_T(\Theta)}{J_r} \\
\frac{k_T(\Theta)}{J_r} \\
\frac{1}{\tau}
\end{bmatrix},
C_t &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\Delta x_t(t) &= \begin{bmatrix}
\Delta \xi(t) \\
\Delta \eta(t) \\
\Delta \xi(t) \\
\Delta \eta(t) \\
\Delta \theta_r(t) \\
\Delta \omega_r(t) \\
\Delta \beta(t)
\end{bmatrix}.
\end{align*}
Then, assuming the measured outputs are $\omega_g(t)$ and $\ddot{y}_t(t)$, and after linearizing the rest of the wind turbine dynamics (1.1)-(1.13), the linear uncertain model is defined as:

$$
\begin{align*}
\Delta \dot{x}_t(t) &= A_t(\Theta)\Delta x_t(t) + B_{wt}(\Theta)\Delta w(t) + B_{ut}\Delta u(t) \\
\Delta y(t) &= C_t\Delta x_t(t) 
\end{align*}
$$

where $\Delta u(t) = \Delta \beta_d(t) = \beta_d(t) - \bar{\beta}$, $\Delta w(t) = \Delta V_w(t) = V_w(t) - \bar{V}_w$, and matrices $A_t(\Theta)$, $B_{wt}(\Theta)$, $B_{ut}$, $C_t$ and linearized system states $\Delta x_t(t)$ are defined in (4.3).

Fig. 4.3 Comparison of the open-loop responses due to change in wind speed and collective blade pitch for the 5 MW reference wind turbine in FAST code and the obtained uncertain model (4.4).

Notice that there is one-to-one correspondence among the values $\bar{V}_w$, $\bar{\omega}_r$ and $\bar{\beta}$ of the operating trajectory $\Theta = (\bar{\omega}_r, \bar{\beta}, \bar{V}_w)$ of the wind turbine (see Fig. 4.1). Therefore, the uncertain model (4.4) can be parametrized by only one variable. In [44, 46] the uncertain wind turbine model was parametrized by $\bar{V}_w$, i.e. $\Theta = (\bar{\omega}_r(\bar{V}_w), \bar{\beta}(\bar{V}_w), \bar{V}_w)$, assuming the
wind mean speed is either obtained from the anemometer installed on the turbine nacelle or estimated from the rotor information. Likewise, the model can also be parametrized by $\bar{\beta}$, as used in [145], that is $\Theta = (\omega_r (\bar{\beta}), \dot{\bar{\beta}}, \dot{V}_w(\bar{\beta}))$. This work parametrizes the wind turbine model using the blade pitch angle $\bar{\beta}$ in order to avoid wind speed measurement. However, note that the state space model (4.4) of the wind turbine still contains 6 linear parameter varying terms defined in (4.2).

A model validation in open-loop is presented in Fig. 4.3, where the responses of the uncertain model are compared to the high-order detailed FAST nonlinear computational model. Since the wind turbine dynamics is dependent on the wind speed and the blade pitch angle, the simulations are presented under variations of $V_w(t)$ and $\bar{\beta}(t)$. Fig. 4.3 shows an acceptable match between the two models. The generator torque was set to its nominal value 43093.55 Nm.

### 4.3 Robust ADR Collective Pitch Control Scheme

The proposed control scheme is an active disturbance rejection LMI-based state feedback collective pitch control (CPC) to regulate the generator speed of the wind turbine in region 3. In the active disturbance rejection framework, all disturbances are assumed to be at the input of the system, to later be estimated and then canceled by using the control law. Therefore, all disturbances of the wind turbine model (4.4) are lumped together and taken at the input using the new variable $z(t)$. Then, the system model is rewritten according to the ADR paradigm as:

$$
\Delta x(t) = A_t(\Theta)\Delta x(t) + B_{\omega}(\Delta u(t) + z(t)) \\
\Delta y(t) = C_t \Delta x(t)
$$

(4.5)

where $z(t)$ lumps together all endogenous and exogenous disturbances, around the operating trajectory, different from those stated in the system model.

### 4.3.1 Disturbance internal model and augmented system

Given a positive integer $p$, the unknown input disturbance $z(t)$ can be modeled by the approximation of its internal model given by

$$
\frac{d^p z(t)}{dt^p} \approx 0.
$$

(4.6)
Consider the following disturbance states, related to (4.6):

\[ x_d(t) = \begin{bmatrix} z(t) & \dot{z}(t) & \cdots & z^{(p-2)}(t) & z^{(p-1)}(t) \end{bmatrix}^T \]  

(4.7)

where its corresponding dynamics is given by

\[ \frac{dx_d}{dt}(t) = A_d x_d(t) + B_d \dot{z}^{(p)}(t) \]

\[ z(t) = C_d x_d(t) \]  

(4.8)

with

\[
A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 \end{bmatrix}^T
\]

(4.9)

where \( x_d(t) \in \mathbb{R}^{p \times 1}, A_d \in \mathbb{R}^{p \times p}, B_d \in \mathbb{R}^{p \times 1}, \) and \( C_d \in \mathbb{R}^{1 \times p}. \)

Now, the disturbance states \( x_d(t) \) can be added to the uncertain system model (4.5) in order to form the following augmented system:

\[ \frac{d}{dt} x(t) = A(\Theta)x(t) + B \Delta u(t) + B_a \dot{z}^{(p)}(t) \]

\[ y(t) = Cx(t) \]  

(4.10)

with

\[
x(t) = \begin{bmatrix} \Delta x_t(t) \\ x_d(t) \end{bmatrix}, \quad A(\Theta) = \begin{bmatrix} A_t(\Theta) & B_{ut} C_d \\ 0 & A_d \end{bmatrix}, \quad B = \begin{bmatrix} B_{ut} \\ B_d \end{bmatrix}
\]

(4.11)

where \( x(t) \in \mathbb{R}^{(p+8)\times 1}, A(\Theta) \in \mathbb{R}^{(p+8)\times(p+8)}, B, B_a \in \mathbb{R}^{(p+8)\times 1}, \) and \( C \in \mathbb{R}^{2 \times (p+8)}. \)

Then, the augmented system (4.10)-(4.11) containing 6 linear parameter varying terms (4.2), can be encapsulated using the maximum and minimum value of each term, by a 64-vertices-polytope, whose system

\[
S = \begin{bmatrix} A(\Theta) & B \\ C & 0 \end{bmatrix}
\]
4.3 Robust ADR Collective Pitch Control Scheme

varies within a fixed polytope of matrices, i.e. \( S \in \mathcal{C}_0 \{ S_1, ..., S_{64} \} \), where \( S_1, ..., S_{64} \) are given vertex systems:

\[
S_1 = \begin{bmatrix}
A_1 & B_1 \\
C_1 & 0
\end{bmatrix}, \ldots, S_{64} = \begin{bmatrix}
A_{64} & B_{64} \\
C_{64} & 0
\end{bmatrix}.
\]

Then, the system \( S \) is a convex combination of the systems \( S_1, S_2, ..., S_{64} \).

The next step is to design an extended state observer for the composite system in (4.10) regarding the approximated internal model given in (4.6). The estimated augmented state vector \( \hat{x}(t) \) contains a real-time estimate of \( x_d(t) \), which is used along with \( C_d \) to recover \( z(t) \).

### 4.3.2 Disturbance estimation

The estimation of the disturbance function \( z(t) \), denoted as \( \hat{z}(t) \), is given by the following observer:

\[
\frac{d}{dt} \hat{x}(t) = A(\Theta)\hat{x}(t) + B\Delta u(t) - K_{obs} (\Delta y(t) - C\hat{x}(t))
\]

\[
\hat{z}(t) = C_z \hat{x}(t)
\]

(4.12)

where \( \hat{x}(t) = [\Delta \xi, \Delta \dot{\xi}, \Delta \hat{\xi}, \Delta \dot{\hat{\xi}}, \Delta \hat{\dot{\xi}}, \Delta \hat{\dot{\hat{\xi}}}, \Delta \hat{\beta}, \Delta \hat{\omega}_r, \Delta \hat{\omega}_g, \Delta \hat{\beta}, \hat{z}, ..., \hat{z}^{(p-1)}]^T \) is the estimated state vector, \( C_z = \begin{bmatrix} 0 & C_d \end{bmatrix} \) and \( K_{obs} \) is the observer gain matrix. The observer (4.12) reconstructs the disturbance \( z(t) \), forcing the state estimation error \( \hat{e}_x(t) = x(t) - \hat{x}(t) \) to converge towards the interior of a disk centered in the origin of the corresponding estimation error phase space, provided the coefficients of the matrix \( K_{obs} \), are chosen in such way that the eigenvalues of the matrix \((A(\Theta) + K_{obs}C)\), or \((A_i + K_{obs}C)\) with \( i = 1, 2, ..., 64 \), are located to the left of the imaginary axis of the complex plane \( s \).

Based on the pole-placement techniques for LPV systems [58, 146, 147], the eigenvalues of the observer can be assigned into corresponding desired regions using LMIs constraints. The LMI problem includes the following optimization objectives:

\[
\text{minimize } \{-\text{trace}(Y)\}
\]

Subject to

\( Y > 0 \)

\( \alpha \)-stability region \( \text{Re}(s) \geq -\alpha_{o2} \):

\[
2\alpha_{o2}Y + A_i^T Y + C^T L + YA_i + L^T C > 0
\]
ADRC approach of Wind Turbines Operating in Full-Load Region

Quadratic cost $J = \int_0^\infty x^T Q_o x + y^T R_o y$:

$$\begin{bmatrix} -A_t^T Y - C^T L + (-A_t^T Y - C^T L)^T & Y & L^T \\ Y & Q_o^{-1} & 0 \\ L & 0 & R_o^{-1} \end{bmatrix} > 0$$

with $(i = 1, 2, ..., 64)$. The goal is to find a quadratic Lyapunov matrix $Y$ and a vector $L$, in order to fulfill all the design objectives for all 64 plants in the polytope. Finally, the observer matrix can be calculated using:

$$K_{obs} = \left( L(Y)^{-1} \right)^T.$$

(4.13)

### 4.3.3 Control scheme

Assume an accurate estimation of the lumped disturbance input signal $z(t)$ given by the observer in (4.12); then, for the open-loop system (4.4), the following state-feedback control law is proposed within the ADR approach:

$$\Delta u(t) = K_c \Delta \hat{x}_t(t) - \hat{z}(t)$$

(4.14)

where $K_c$ is the control gain vector and $\hat{x}_t(t)$ is provided by the observer in (4.12). The control law (4.14) rejects the disturbance $z(t)$ (within the observer’s bandwidth), and forces the system to be along the optimal trajectory of the wind turbine in region 3, provided the coefficients of the vector $K_c$, are chosen in such way that the eigenvalues of the matrix $(A_t(\Theta) + B_{ut}K_c)$, or $(A_{ti} + B_{ut}K_c)$ with $i = 1, 2, ..., 64$, are located to the left of the imaginary axis of the complex plane $s$.

Then, following a dual procedure as in the observer design, we can apply pole-placement techniques in order to assign the eigenvalues of the closed-loop system into corresponding desired regions using LMIs constraints. The LMI control problem has been divided in two cases which are: an $H_\infty$ minimization, and a LQR problem. In order to limit the bandwidth of the control system, each control problem is combined with a pole-placement constraint $Re(s) \geq -\alpha_2$. Table 4.3 summarizes the control cases. If there exist a symmetric positive definite matrix $P$ and a vector $Z$ which fulfill all the design objectives for all 64 plants in the polytope, then the control gain can be calculated using:

$$K_c = Z(P)^{-1}.$$

(4.15)
Table 4.3 LMI Control problems of the proposed ADR control scheme.

<table>
<thead>
<tr>
<th>minimize ($\sigma$)</th>
<th>maximize (trace($P$))</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P &gt; 0$</td>
<td>$P &gt; 0$</td>
</tr>
<tr>
<td>$\alpha$ - stability region (Re(s) $\geq -\alpha_c$) :</td>
<td>$\alpha$ - stability region (Re(s) $\geq -\alpha_c$) :</td>
</tr>
<tr>
<td>$2\alpha_c P + A_{ii} P + B_{uu} Z + PA_{ii}^T + Z^T B_{uu}^T &gt; 0$</td>
<td>$2\alpha_c P + A_{ii} P + B_{uu} Z + PA_{ii}^T + Z^T B_{uu}^T &gt; 0$</td>
</tr>
<tr>
<td>$H_m$ performance (from $\Delta V_w$ to $\Delta \omega$) :</td>
<td>Quadratic cost $J = \int_0^\infty (\Delta x_t^T Q_c \Delta x_t + \Delta u^T R_c \Delta u)$ :</td>
</tr>
<tr>
<td>$\begin{bmatrix} A_{ii} P + B_{uu} Z + PA_{ii}^T + Z^T B_{uu}^T &amp; B_{uu} \ C_t P \ 0 &amp; -\sigma I \end{bmatrix} &lt; 0$</td>
<td>$\begin{bmatrix} -A_{ii} P - B_{uu} Z - (A_{ii} P + B_{uu} Z)^T &amp; P &amp; Z^T \ P &amp; Q_c^{-1} &amp; 0 \ Z &amp; 0 &amp; R_c^{-1} \end{bmatrix} &gt; 0$</td>
</tr>
</tbody>
</table>

4.3.4 Results

In this section, the simulations carried out in the FAST code to assess the performance of the proposed ADR CPC scheme to operate a 5 MW wind turbine in full-load region are described.

Baseline controller

The baseline collective pitch Gain Scheduling (GS) PI controller is defined as [78]:

$$\Delta \beta_d(t) = K_P(\beta) N_g \omega_r(t) + K_I(\beta) \int_0^t N_g \omega_r(t) dt,$$

(4.16)

with,

$$K_P(\beta) = \frac{2 \left( J_r + N_s^2 I_g \right) \omega_N \zeta_{pi} \omega_{pi}}{N_g \left[ -\frac{\partial P}{\partial \beta} \right]_{\beta=0} G_k(\beta)}$$

(4.17)

$$K_I(\beta) = \frac{\left( J_r + N_s^2 I_g \right) \omega_N \omega_{pi}^2}{N_g \left[ -\frac{\partial P}{\partial \beta} \right]_{\beta=0} G_k(\beta)}$$

(4.18)

$$G_k(\beta) = \frac{1}{1 + \frac{\beta(t)}{\beta_k}}$$

(4.19)

where, $\zeta_{pi} = 0.7$ is the desired damping ratio, $\omega_{pi} = 0.6$ rad/s is the desired natural frequency, $\beta_k = 0.1099$ rad and $\partial P/\partial \beta|_{\beta=0} = -25.52 \times 10^6$ watt/rad.
ADR controller design

According to sections (4.3.2) and (4.3.3), three ADR CPC schemes are proposed. The first ADRC scheme (ADR CPC1) assumes the disturbance internal model approximation as \( \frac{dz(t)}{dt} \approx 0 \) i.e. \( p = 1 \), and given \( Q_o, R_o \), and the pole-placement constraint \( Re(s) \geq -\alpha_o^2 \) a robust LQ observer is designed for all 64 plants of the uncertain model of the wind turbine. Then, a robust state-feedback control gain is also designed, for all 64 plants, to fulfill a linear quadratic cost given the parameters \( Q_c, R_c \) and the pole-placement constraint \( Re(s) \geq -\alpha_c^2 \).

The third scheme (ADR CPC3) is designed using the same observer as in (ADR CPC1), but the control gain is calculated by minimizing the \( H_\infty \) performance from \( \Delta V_w \) to \( \Delta \omega_g \) and the pole-placement constraint \( Re(s) \geq -\alpha_c^2 \). The second scheme (ADR CPC2) assumes the disturbance internal model approximation as \( \frac{d^2z(t)}{dt^2} \approx 0 \) i.e. \( p = 2 \), and given \( Q_o, R_o \), and the pole-placement constraint \( Re(s) \geq -\alpha_o^2 \) a robust LQ observer is designed for the uncertain model. The control gain is calculated, as in the case of (ADR CPC2), by minimizing the \( H_\infty \) performance from \( \Delta V_w \) to \( \Delta \omega_g \) and the pole-placement constraint \( Re(s) \geq -\alpha_c^2 \).

The design parameters of each ADRC scheme are summarized in Table 4.4. Note that the weighting matrices \( Q_o \) of each observer have been tuned considering more attention to the estimation of the disturbance signal \( z(t) \) (9th state). The Fig. 4.4 shows the results of the eigenvalue location of each observer and closed-loop control scheme for the 64-vertices-polytope model of the 5 MW wind turbine.

[Diagrams of eigenvalue location for each scheme]

Fig. 4.4 Eigenvalue location of each ADR CPC scheme for the 64-vertices-polytope system.
Table 4.4 List of the design parameters of each ADR collective pitch controller.

<table>
<thead>
<tr>
<th>ADR Collective Pitch Controller</th>
<th>ADR Observer and Control Design Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPC1</td>
<td>( Q_o = \begin{bmatrix} 2 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 2 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 2 \end{bmatrix} ), ( R_o = \begin{bmatrix} 16 &amp; 0 \ 0 &amp; 16 \end{bmatrix} ), ( \alpha_o^2 = 15 ), ( Q_c = \begin{bmatrix} 12 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 12 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 12 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 12 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} ), ( R_c = 16 ), ( \alpha_c^2 = 4 )</td>
</tr>
<tr>
<td>CPC2</td>
<td>( Q_o = \begin{bmatrix} 0.02 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0.01 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0.02 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0.01 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0.01 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0.01 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0.01 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0.01 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 3 \end{bmatrix} ), ( R_o = \begin{bmatrix} 36367 &amp; 0 \ 0 &amp; 36367 \end{bmatrix} ), ( \alpha_o^2 = 9 ), ( \alpha_c^2 = 3 )</td>
</tr>
<tr>
<td>CPC3</td>
<td>( Q_o = \begin{bmatrix} 2 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 2 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 2 \end{bmatrix} ), ( R_o = \begin{bmatrix} 16 &amp; 0 \ 0 &amp; 16 \end{bmatrix} ), ( \alpha_o^2 = 15 ), ( \alpha_c^2 = 4 )</td>
</tr>
</tbody>
</table>
Stepwise wind profile

For this test, a rise/fall wind profile is used as shown in Fig. 4.5. Initially, the horizontal wind speed is kept constant at 17 m/s, and then it is changed from 17 to 24 m/s between 63 and 65 sec. After that, the wind speed is decreased from 24 to 14 m/s between 155 and 160 sec. This profile let evaluate the behavior of each control system under extreme conditions.

Fig. 4.5 Stepwise wind speed profile.

The closed-loop response comparison between the baseline GS PI controller (red line) and each proposed ADR control scheme is shown in Fig. 4.6. It can be observed that the ADR schemes achieve more effective regulation of the generator speed against sudden changes in the wind speed (wind gusts). This has a significant effect on the generated power specially at falling edge as seen in the figure at 160 sec. A better regulation of the generator speed is found at the cost of slightly faster blade pitch angle changes. The figure also shows the disturbance estimation 〈z(t)〉 of each ADR CPC scheme, which illustrates how the estimation of z(t) dominantly affects the control signal. In fact, the injection of the disturbance estimation 〈z(t)〉 into the control law has eliminated the steady-state error on the generator speed. In addition, note that the maximum and minimum Out-of-Plane (OoP) and flap-wise bending moments of rotor blades have been attenuated compared to the baseline controller. Some oscillations in the Out-of-Plane and flap-wise bending moments are observed, which are mainly generated due to periodic effects such as tower shadow and rotor misalignment. These oscillations are controlled by means of IPC (Individual Pitch Control) in the next section.
Fig. 4.6 Simulation results comparing the 3 proposed ADR control schemes vs the baseline controller using a wind rise/fall profile.
Turbulent wind profile

In this test, the wind profile (see Fig. 4.7) consists of a 250 sec realistic 3-D wind speed field generated using Class A Kaimal turbulence spectra with TurbSim [148]. This profile has a mean value of 17 m/s at the hub height, turbulence intensity of 25% and normal IEC (International Electrotechnical Commission) turbulence type.

The Fig. 4.7 shows the closed-loop responses obtained with each proposed ADR scheme. The figure shows that the 3 proposed robust ADR control schemes provide better speed and power regulation than the baseline controller while maintaining the blade bending moments at low levels.

The Table 4.5 shows some statistic data of both the angular speed and generated power. Note that the standard deviations have been reduced and also the maximum angular generator speeds have also been reduced. This helps prevent the mechanical overload on the drive-train, and thus increasing the life time of the system. Moreover, the maximum power has been significantly reduced. This prevents false shutdowns of the generator due to overloading.
Fig. 4.8 Simulation results of 3 proposed ADR CPC schemes vs the baseline controller under a 25% turbulence intensity wind profile.

Table 4.5 Speed and power data analysis for turbulent profile.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>ADR CPC1</th>
<th>ADR CPC2</th>
<th>ADR CPC3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Generator speed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(% of rated speed)</td>
<td>Max.</td>
<td>132.64%</td>
<td>107.01%</td>
<td>108.85%</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>100.35%</td>
<td>99.86%</td>
<td>99.95%</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>55.04 rpm</td>
<td>27.85 rpm</td>
<td>30.64 rpm</td>
</tr>
<tr>
<td><strong>Electric power</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(% of rated power)</td>
<td>Max.</td>
<td>132.63%</td>
<td>107.01%</td>
<td>108.85%</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>100.35%</td>
<td>99.86%</td>
<td>99.95%</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>234.47 kW</td>
<td>118.67 kW</td>
<td>130.56 kW</td>
</tr>
</tbody>
</table>
4.4 ADR Individual Pitch Control approach for Periodic Disturbances

Wind turbines are mainly disturbed by two effects named wind shear and tower shadow. The term wind shear is used to describe the variation of wind speed with height, while the term tower shadow describes the redirection of wind due to the tower structure [79]. Thus, even for a constant wind speed at a particular height, a turbine blade would encounter variable wind as it rotates. Torque pulsations are observed due to the periodic variations of wind speed experienced at different locations [81]. Additionally, such periodic variations in the aerodynamic torque contribute significantly decreasing the life-time of each blade due to fatigue accumulation [82]. These variations lead to 1P (once per revolution), 2P and 4P large components in the blade loads (rotating frame of reference), and 0P and 3P components on the fixed structure (non-rotating frame) such as nacelle and tower [58]. This has motivated the development of blade IPC methodologies, many of which employ the Coleman transformation (or MBC transformation) to simplify the controller design process [66]. In IPC design, the Coleman transformation expresses the states, inputs and outputs of the periodic LTV wind turbine model in a nonrotating coordinate frame. The MBC transformation does not directly result in an LTI system, but the MBC approach usually yields a model that is weakly periodic and averaging of system matrices can result in a LTI model of sufficient accuracy [124].

4.4.1 System Model for IPC

In the IPC scheme the Coleman transformation is used [66]. According to this, the action of the flap per blade could be independently controlled. In general, the flap-wise bending moments of the three blades, $M_{y1}(t)$, $M_{y2}(t)$ and $M_{y3}(t)$ are first transformed into the fixed frame of reference using the inverse Coleman transformation, yielding the static hub yaw-wise moment $M_{yaw}(t)$ and tilt-wise moment $M_{tilt}(t)$, respectively:

\[
M_{y0}(t) = \frac{1}{3} \sum_{b=1}^{3} M_{yb}(t)
\]
\[
M_{yaw}(t) = \frac{2}{3} \sum_{b=1}^{3} M_{yb}(t) \sin \theta_b(t)
\]
\[
M_{tilt}(t) = \frac{2}{3} \sum_{b=1}^{3} M_{yb}(t) \cos \theta_b(t)
\]

(4.20)
where $M_{y0}(t)$ is the average blade root flap-wise bending moment, $\theta_b(t)$ is the azimuth angle of each blade as shown in (4.21):

$$\theta_b(t) = \theta_r(t) + \frac{2\pi}{3}(b-1), \ b = 1, 2, 3,$$

with $\theta_r(t)$ is the azimuth of the first blade and $\theta_r(t) = 0$ stands for the first blade to stand vertically up. Then, the inverse Coleman transform is defined as:

$$
\begin{bmatrix}
M_{y0}(t) \\
M_{tilt}(t) \\
M_{yaw}(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{3} \\
\frac{2}{3} \cos \theta_r(t) \\
\frac{2}{3} \sin \theta_r(t)
\end{bmatrix}
\begin{bmatrix}
\frac{1}{3} \\
\frac{2}{3} \cos \left(\theta_r(t) + \frac{2\pi}{3}\right) \\
\frac{2}{3} \sin \left(\theta_r(t) + \frac{2\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
\frac{1}{3} \\
\frac{2}{3} \cos \left(\theta_r(t) + \frac{4\pi}{3}\right) \\
\frac{2}{3} \sin \left(\theta_r(t) + \frac{4\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
M_{y1}(t) \\
M_{y2}(t) \\
M_{y3}(t)
\end{bmatrix}.
$$

(4.22)

Also, the individual pitch angles $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are related to the tilt and yaw pitch angles, $\beta_{tilt}(t)$ and $\beta_{yaw}(t)$, respectively, via the Coleman transform:

$$
\begin{bmatrix}
\beta_1(t) \\
\beta_2(t) \\
\beta_3(t)
\end{bmatrix} =
\begin{bmatrix}
1 & \cos \theta_r(t) & \sin \theta_r(t) \\
1 & \cos \left(\theta_r(t) + \frac{2\pi}{3}\right) & \sin \left(\theta_r(t) + \frac{2\pi}{3}\right) \\
1 & \cos \left(\theta_r(t) + \frac{4\pi}{3}\right) & \sin \left(\theta_r(t) + \frac{4\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
\beta_c(t) \\
\beta_{tilt}(t) \\
\beta_{yaw}(t)
\end{bmatrix},
$$

(4.23)

where $\beta_c(t) = \bar{\beta}(t)$ is the collective blade pitch angle or averaged blade-pitch angle demand. Fig. 4.9 details the general control structure for Collective Pitch Control (CPC) and Individual Pitch Control (IPC). Note that the control action of the IPC is added to that of the CPC scheme.

Fig. 4.9 General scheme of the IPC and CPC strategies via the Coleman Transform.
On the other hand, the linearized dynamics of the rotating frame of reference consists of three identical transfer functions $G_{BM}$, relating the blade pitch angle demands $\beta_{1,2,3}(t)$ and the blade root flap-wise bending moments $M_{y1,y2,y3}(t)$ [66]:

$$
G_{BM}(s) = \frac{1}{\tau s + 1} \frac{dM_{\text{flap}}}{d\beta} \frac{(2\pi f_b)^2}{s^2 + D_b 2\pi f_b s + (2\pi f_b)^2 s^2 + 2\pi (f_h + f_l) s + 4\pi^2 f_h f_l} \tag{4.24}
$$

where $\tau$ is actuator pitch time constant, $\frac{dM_{\text{flap}}}{d\beta}$ is the variation of blade flap-wise bending moment with respect to pitch angle, $f_b$ is the natural frequency of first blade flap-wise, $D_b$ is the blade aerodynamic damping ratio, $f_h$ is the band-pass filter high corner frequency and $f_l$ is the band-pass filter low corner frequency. Then, applying the Coleman transform to system (4.24) (see [66]), a fixed-frame coordinate system is found (see Fig. 4.10) whose dynamics is described by:

$$
\begin{bmatrix}
M_{\text{tilt}}(s) \\
M_{\text{yaw}}(s)
\end{bmatrix} =
\begin{bmatrix}
G_{BM}(s+j\omega_r) + jG_{BM}(s-j\omega_r) \\
-G_{BM}(s+j\omega_r) - jG_{BM}(s-j\omega_r)
\end{bmatrix}
\begin{bmatrix}
G_{BM}(s+j\omega_r) - G_{BM}(s-j\omega_r) \\
G_{BM}(s+j\omega_r) + jG_{BM}(s-j\omega_r)
\end{bmatrix}
\begin{bmatrix}
\beta_{\text{tilt}}(s) \\
\beta_{\text{yaw}}(s)
\end{bmatrix}
\tag{4.25}
$$

Then, based on the fixed frame transformed system (4.25) and according to the conventional IPC Coleman transform scheme (for 1P frequency), a simple inner control loop with $G_c(s) = \frac{K_0}{s}$ is used for each output $M_{\text{yaw}}$ and $M_{\text{tilt}}$. This inner loop allows obtaining a simplified second-order dominant dynamics whose main objective is to facilitate the design of the ADR/IPC scheme by keeping a low-order control approach. Thus, the ADR/IPC scheme (see Fig. 4.11) is designed based on the simplified dominant closed-loop dynamics of the inner control loop. The block diagram of the general control approach is shown in Fig. 4.11, where the light red block represents the proposed ADR scheme. Note that the $M_{\text{tilt}}$ and $M_{\text{yaw}}$ control loops are identical since $C^\text{PI}_{11} = C^\text{PI}_{22}$. Therefore, the ADR control scheme is proposed for the tilt control loop, but it is also applicable to the yaw control loop.
The system from $M_{\text{tilt}}^*(t)$ to $M_{\text{tilt}}(t)$, can be dominantly described as:

$$
\dot{x}_{\text{tilt}} = A^{\text{tilt}} x_{\text{tilt}} + B^{\text{tilt}} M_{\text{tilt}}^* + B^{\text{tilt}} d_{\text{tilt}}
$$

$$
M_{\text{tilt}} = C^{\text{tilt}} x_{\text{tilt}}
$$

where, $x_{1}^{\text{tilt}} = M_{\text{tilt}}(t)$, $x_{2}^{\text{tilt}} = \dot{M}_{\text{tilt}}(t)$, $x_{\text{tilt}} = \begin{bmatrix} x_{1}^{\text{tilt}} & x_{2}^{\text{tilt}} \end{bmatrix}^T$, $\{a_0, a_1, b_0\}$ are constants that define a dominant second order system,

$$
A^{\text{tilt}} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, \quad B^{\text{tilt}} = \begin{bmatrix} 0 \\ b_0 \end{bmatrix}, \quad C^{\text{tilt}} = \begin{bmatrix} 1 & 0 \end{bmatrix},
$$

and $d_{\text{tilt}}(t)$ represents all coupling dynamics and exogenous disturbances affecting the system, in particular those disturbances at the frequencies 0P and 3P.
4.4.2 ADR Observer-based Control Scheme

The proposed control scheme is based on the ADR paradigm where some frequency components of the unified disturbance \( d(t) \) of the system (4.26) are estimated and then rejected by means of an observer-based control law. Then, the ADRC scheme is composed by two parts: the observer and the control law. The purpose of the observer is to estimate the disturbance signal \( d(t) \) and system states. The control law provides disturbance rejection based on the disturbance signal estimation and assures a dominant closed-loop system dynamics.

This disturbance can be a complex signal to estimate depending on how many components of the signal should be precisely estimated. In this case, two components of \( d(t) \) are important, the 0P and 3P frequency components. So, in order to provide a precise estimate of each component, the internal model of the observer must contain the annihilator of each component of interest: (a) the annihilator of a constant signal and (b) the annihilator of a sinusoidal signal with frequency 3P. Therefore, an internal model approximation of the disturbance \( d(t) \), can be stated as:

\[
\varphi(t) = (\ddot{d}(t) + \omega_{3p}^2 d(t)) \approx 0, \quad (4.27)
\]

where \( \omega_{3p} \) is the 3P frequency affecting the fixed frame of the structure. Note that (4.27) contains the annihilator of a constant signal, \( \dot{d}(t) \), and the annihilator of a sinusoidal signal with frequency 3P, \( (\ddot{d}(t) + \omega_{3p}^2 d(t)) \), then the internal model (4.27) provides disturbance rejection when \( d(t) = d_0 + \sin(\omega_{3p} t) \), with \( d_0 \) any finite constant.

Consider the following disturbance states, related to (4.27),

\[
z(t) = \left[ \begin{array}{c} d(t) \\ \dot{d}(t) \\ \ddot{d}(t) \end{array} \right] = \left[ \begin{array}{c} z_1(t) \\ z_2(t) \\ z_3(t) \end{array} \right] = z(t) \quad (4.28)
\]

where their corresponding dynamics is given by:

\[
\dot{z}(t) = A_d z(t) + B_d \varphi(t) \\
\dot{d}(t) = C_d z(t) \quad (4.29)
\]

with

\[
A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_{3p}^2 & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},
\]

where \( z(t) \in \mathbb{R}^{3 \times 1} \), \( A_d \in \mathbb{R}^{3 \times 3} \), \( B_d \in \mathbb{R}^{3 \times 1} \) and \( C_d \in \mathbb{R}^{1 \times 3} \).
Then, it is possible to augment the system (4.26) with the unknown input disturbance state vector \( z(t) \); thus,

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{z}(t)
\end{bmatrix}
= \begin{bmatrix}
A & BC_d \\
0 & A_d
\end{bmatrix}
\begin{bmatrix}
x(t) \\
z(t)
\end{bmatrix}
+ \begin{bmatrix}
B \\
0
\end{bmatrix} M^*(t) + \begin{bmatrix}
0 \\
B_d
\end{bmatrix} \varphi(t) \tag{4.30}
\]

\[
M(t) = \begin{bmatrix}
C \\
C_a
\end{bmatrix}
\begin{bmatrix}
x(t) \\
z(t)
\end{bmatrix}
\tag{4.31}
\]

where \( A_d \in \mathbb{R}^{5 \times 5}, B_d, B_{da} \in \mathbb{R}^{5 \times 1} \) and \( C_a \in \mathbb{R}^{1 \times 5} \). Based on (4.26), (4.27), and (4.31), the following ADR observer-based control scheme is proposed.

**Theorem 4.1 (Disturbance \( d_{\text{tilt}} \) observer)** The estimation of the disturbance function \( d_{\text{tilt}}(t) \), denoted as \( \hat{d}_{\text{tilt}}(t) \), is given by the following GPI observer:

\[
\begin{bmatrix}
\dot{x}_{\text{tilt}}(t) \\
\dot{z}_{\text{tilt}}(t)
\end{bmatrix} = \begin{bmatrix}
A^\text{tilt}_{a} & x_{\text{tilt}}(t) \\
0 & x_{\text{tilt}}(t)
\end{bmatrix}
+ \begin{bmatrix}
B^\text{tilt}_{a} \\
0
\end{bmatrix} M_{\text{tilt}}^*(t) + \begin{bmatrix}
L^\text{tilt}_{a} \\
0
\end{bmatrix} C^\text{tilt}_{a} x_{\text{tilt}}(t)
\]

\[
\hat{d}_{\text{tilt}}(t) = \begin{bmatrix}
0 \\
C^\text{tilt}_{d a}
\end{bmatrix} \dot{z}_{\text{tilt}}(t) \tag{4.32}
\]

where \( \dot{x}_{\text{tilt}}(t) = \begin{bmatrix}
\dot{x}_{\text{tilt}}(t) \\
\dot{z}_{\text{tilt}}(t)
\end{bmatrix}^T \) is the estimated state vector and \( L^\text{tilt} \) is the observer gain vector and \( C^\text{tilt}_{d a} = \begin{bmatrix}
0 \\
C^\text{tilt}_{d}
\end{bmatrix} \). The observer (4.32) asymptotically and exponentially reconstructs the disturbance \( d_{\text{tilt}}(t) \), forcing the state estimation error \( \varepsilon_{x_{\text{tilt}}} \) to converge towards the interior of a disk centered in the origin of the corresponding estimation error phase space, provided the coefficients of the vector \( L^\text{tilt} \), are chosen in such way that the eigenvalues of the matrix \( [A^\text{tilt}_{a} - B^\text{tilt}_{a} K_{\text{tilt}}] \) are located to the left of the imaginary axis of the complex plane \( s \).

**Proof.** See Appendix A.5. ■

**Theorem 4.2 (ADR Control Law for Tilt-Moment)** Given an accurate estimation of \( d_{\text{tilt}}(t) \) and \( x_{\text{tilt}}(t) \), the following control law is proposed:

\[
M^*_{\text{tilt}}(t) = -K_{\text{tilt}} x_{\text{tilt}}(t) - \hat{d}_{\text{tilt}}(t) \tag{4.33}
\]

where \( K_{\text{tilt}} \) is the control gain, \( \hat{d}_{\text{tilt}}(t) \) and \( \dot{x}_{\text{tilt}}(t) \) are provided by the GPI observer given in Theorem 4.1. Such control law asymptotically and exponentially forces the bending moment \( M_{\text{tilt}}(t) \) to converge towards a small vicinity of zero, and rejects the disturbances affecting the WECS in both 0P and 3P periodic components of the tilt-wise moment in the fixed frame of the structure, provided that the matrix \( (A^\text{tilt} - B^\text{tilt} K_{\text{tilt}}) \) is Hurwitz.
Proof. See Appendix A.6.

The ADR observer-based control scheme for the yaw-moment control loop is not shown due to the disturbance observer of $d_{yaw}(t)$ and its corresponding control law are both analogous to theorems 4.1 and 4.2, respectively. The Fig. 4.12 shows the block diagram of the yaw and tilt control loops.

### 4.4.3 Spatial ADR Observer-based Control Scheme

Several mechatronic rotary systems are exposed to many kinds of disturbances. But, due to the nature of the these systems, periodic disturbances are one of the most common type of disturbances. They appear mainly because of eccentricities, axis unbalance, mass non-uniformity, couplings or pulsating torques.

To deal with the periodic disturbance rejection problem, well established control strategies as Repetitive Control (RC) [149] and Adaptive Feedforward Cancellation (AFC) [150] have proven to be very effective. These strategies are based on the Internal Model Principle (IMP) [151], which states that in order to track/reject an exogenous signal, the model of such signal
must be included in the control loop. Both RC and AFC assume the exact knowledge of the signal frequency since this information is included in the internal model of the signal. If the speed of the mechatronic system remains constant, the fundamental frequency of the disturbance is also constant and the above mentioned techniques can be applied successfully. However, if the rotational speed changes, the frequency would change proportionally which cause that RC drastically loses its performance [152, 153].

To allow RC operate properly at varying speed, some modifications can be made: 1) include an adaptive system in which the frequency of the internal model varies according to the signal frequency (speed). In this way, the system needs a frequency estimator and becomes a variable structure system [154, 155] which complicates the stability analysis. 2) Employ high order internal models to provide robustness against frequency changes, known as High Order Repetitive Control (HORC) [156]. The main drawbacks of HORC are that the order of the controller is very large and only small frequency changes are allowed. 3) Implement a digital system that adjust the sampling frequency according to the speed changes [157], in order to keep constant the number of samples per period of the disturbance signal. This allows larger frequency changes but involves a more complex stability analysis since the control system is a Linear Time Varying system [158].

All above mentioned strategies are formulated in time domain; however, the Spatial Repetitive Control approach presented in [143, 159] uses the angular position instead of time as the independent variable. The main idea behind spatial RC is that the disturbances generated in mechatronic systems are position dependent disturbances (those coming from eccentricities, axis unbalance, mass non-uniformity, coupling torques, etc.). Thus, the rotation of the system generates a disturbance that in the time domain has a frequency that varies proportionally with angular speed but in the spatial domain the frequency of disturbance remains invariant. As a consequence, a RC strategy in which the frequency of the disturbance is assumed fixed can be applied if the position domain is used instead of time.

In this way, when a wind turbine experiences changes in wind speed due to wind shear, tower shadow, yaw misalignment and turbulence, such variations lead to periodic components (bending moments) in the blade loads whose fundamental frequency is dependent on the rotor angular speed. Therefore, under small variations of the rotor speed, well established control strategies as RC and AFC could be successfully applied (see recent works of RC applied to wind turbines [63–65, 160]). However, wind turbines are always exposed to large and unknown disturbances, as consequence the fundamental frequency of the periodic disturbance changes with the rotor angular speed. Fig. 4.13 shows how a small deviation of 2% on the rotor nominal speed causes variations on the frequencies of each periodic load moment (see time-domain responses in the first row of Fig. 4.13). On the other hand, the
This section presents a linear Active Disturbance Rejection Individual Pitch Control scheme in spatial-domain to reject the main periodic load disturbances of wind turbines operating in region 3. The linear nature of the proposed scheme provides itself a high level of simplicity compared with the above mentioned proposals since this constitutes a completely linear design and does not need any adaptive mechanism or frequency estimator.

**Spatial-domain system model**

This section presents the system transformation from time domain to spatial domain. The system into consideration is the dynamics of the non-rotating frame of reference, from $M_{\text{tilt}}^*(t)$ to $M_{\text{tilt}}(t)$, whose dynamics (4.26) can be rewritten using the following differential equation:

$$\ddot{M}_{\text{tilt}}(t) + a_1 M_{\text{tilt}}(t) + a_0 M_{\text{tilt}}(t) = b_0 M_{\text{tilt}}^*(t) + b_0 d_{\text{tilt}}(t).$$  \hspace{1cm} (4.34)
As described in [161], the relation between time and space results in:

\[ \theta = f(t) = \int_0^t \omega(\tau) d\tau + \theta(0), \]

with \( \theta \) the angular position in revolutions and \( \omega(t) \) the angular speed in rev/s. Hence, \( \theta \) is the azimuth rotor angle of the wind turbine in revolutions, and the \( \omega(t) \) is the rotor angular speed in rev/s. The condition, \( \omega(t) = \frac{d\theta}{dt} > 0 \), must be accomplished in order to assure the existence of the inverse function \( t = f^{-1}(\theta) \). Thus, a variable \( g \) defined in time and space domain is related by \(^2\):

\[ g(\theta) = g(f^{-1}(\theta)). \]

Therefore, the transformation from time domain to spatial domain is defined by [143]:

\[ \frac{d}{dt} g(t) = \frac{d\theta}{dt} \frac{dg(\theta)}{d\theta} = \omega(\theta) \frac{dg(\theta)}{d\theta}. \quad (4.35) \]

Thus, applying (4.35) to (4.34), results the non-linear position invariant system:

\[ \frac{d^2 M_{\text{tilt}}(\theta)}{d\theta^2} = \frac{b_0}{[\omega(\theta)]^2} (M_{\text{tilt}}^*(\theta) + d_{\text{tilt}}(\theta)) + \frac{1}{\omega(\theta)} \left[ -a_0 - a_1 \frac{dM_{\text{tilt}}(\theta)}{d\theta} - \left( \frac{dM_{\text{tilt}}(\theta)}{d\theta} \right)^2 \right] \]

\[ (4.36) \]

Furthermore, a variable change \( \psi(\theta) = \frac{M_{\text{tilt}}^*(\theta)}{[\omega(\theta)]^2} \) applied to (4.36), which constitutes a partial feedback linearization, allows obtaining a simplified linear model:

\[ \frac{d^2 M_{\text{tilt}}(\theta)}{d\theta^2} = \kappa \psi(\theta) + \xi_{-1}(\theta) \]

\[ (4.37) \]

with \( \kappa = b_0 \) the system input gain and

\[ \xi_{-1}(\theta) = \frac{1}{\omega(\theta)} \left[ -a_0 - a_1 \frac{dM_{\text{tilt}}(\theta)}{d\theta} - \left( \frac{dM_{\text{tilt}}(\theta)}{d\theta} \right)^2 \right] + \frac{b_0}{[\omega(\theta)]^2} d_{\text{tilt}}(\theta) \]

an unified disturbance term. It is important to note that the disturbance term \( \xi_{-1}(\theta) \) groups the non-linear part of the spatial model, uncertainties and external disturbances; and the obtained simplified structure in (4.37) is the basis to construct the ADRC scheme in spatial-domain.

The model (4.37) constitutes a continuous spatial model. However, the practical implementation requires a spatial clock to run the control strategy in spatial domain. An incremental encoder is used for this purpose which provides a discrete clock to the system.

\(^{2}\)For the sake of clarity, the spatial-domain notations will be denoted by a lower bar.
Therefore, a spatial discretization is needed and the discretization step depends on the number of Pulses Per Revolution (PPR) of the incremental encoder.

To obtain a discrete version of the model (4.37) an Euler derivative approximation is used (this approximation is consequent with the denominated delta operator in [162]):

\[
\frac{dM_{tilt}(\theta)}{d\theta} \approx \frac{(q-1)}{\Delta_\theta} M_{tilt}(k), \quad \frac{d^2M_{tilt}(\theta)}{d\theta^2} \approx \frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k)
\]

with \( q \) the advance operator, \( k \) the spatial sampling step and \( \Delta_\theta \) the inverse of the incremental encoder PPR. In this way, the discrete representation of (4.37) is given by

\[
\frac{d^2M_{tilt}(\theta)}{d\theta^2}\Bigg|_{\theta=k\Delta_\theta} = \kappa_M(\theta)|_{\theta=k\Delta_\theta} + \xi_1(\theta)|_{\theta=k\Delta_\theta},
\]

\[
\frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k) + \frac{d^2M_{tilt}(\theta)}{d\theta^2}\Bigg|_{\theta=k\Delta_\theta} - \frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k) = \kappa_M(\theta)|_{\theta=k\Delta_\theta} + \xi_1(\theta)|_{\theta=k\Delta_\theta} - \left[ \frac{d^2M_{tilt}(\theta)}{d\theta^2}\Bigg|_{\theta=k\Delta_\theta} - \frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k) \right],
\]

which renders the following simplified discrete spatial domain model

\[
\frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k) = \kappa_M(k) + \xi(k),
\]

(4.38)

with

\[
\xi(k) = \xi_1(k)|_{\theta=k\Delta_\theta} - \left[ \frac{d^2M_{tilt}(\theta)}{d\theta^2}\Bigg|_{\theta=k\Delta_\theta} - \frac{(q-1)^2}{\Delta_\theta^2} M_{tilt}(k) \right].
\]

(4.39)

It is important noticing that \( \xi(k) \) takes into account the errors caused by using the Euler approximation method under a suitable selection of the sampling interval. The function \( \xi(k) \) can be denoted as an additive disturbance function without defining any particular structure on it.

Finally, defining

\[
\mathcal{A}_1(k) = M_{tilt}(k),
\]

(4.40)

\[
\mathcal{A}_2(k) = \left( \frac{q-1}{\Delta_\theta} \right) M_{tilt}(k),
\]

(4.41)
the following spatial state-space representation is obtained

$$
\begin{bmatrix}
x_1(k+1) \\
x_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 & \Delta_\theta \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k)
\end{bmatrix} +
\begin{bmatrix}
0 & \Delta_\theta \\
0 & \Delta_\theta \kappa
\end{bmatrix}
\xi(k) +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
v(k)
$$

(4.42)

Unified disturbance $\xi(k)$ observer

The purpose of the observer is to estimate the disturbance signal $\xi(k)$. This disturbance can be a complex signal to estimate depending on how many components of the signal should be precisely estimated. In this case, two components of $\xi(k)$ are important, the 0P and 3P frequency components. So, in order to provide a precise estimate of each component, the internal model of the observer must contain the annihilator of each component of interest: (a) the annihilator of a constant signal and (b) the annihilator of a sinusoidal signal with frequency 3P. Therefore, an internal model approximation of the signal $\xi(k)$, can be stated as:

$$
\varphi(k) = (q - 1) \left( q^2 - 2 \cos(\omega_{1s} \Delta_\theta) q + 1 \right) \xi(k) \approx 0,
$$

(4.43)

where $\omega_{1s} = 6\pi$ rad/s is the 3P frequency affecting the fixed frame of the structure in spatial-domain. Note that $(q - 1) \xi(k)$ corresponds to the annihilator of a constant signal and $(q^2 - 2 \cos(\omega_{1s} \Delta_\theta) q + 1) \xi(k)$ corresponds to the annihilator of a sinusoidal signal with frequency $\omega_{1s}$, then the internal model (4.43) provides disturbance rejection when $\xi(k) = d_0 + \sin(\omega_{1s} k)$, with $d_0$ any finite constant.

In order to propose the observer, the following assumptions are stated:

A1 Disturbance function, $\xi(k)$, is unknown while the input gain, $\kappa$, is known.

A2 Sampling interval $\Delta_\theta$ is sufficiently small to achieve the required accuracy using the Euler discretization method.

A3 There exists a finite constant $K_\xi$, such that,

$$
\sup_k \left| (q - 1) \left( q^2 - 2 \cos(\omega_{1s} \Delta_\theta) q + 1 \right) \xi(k) \right| \leq K_\xi
$$

(4.44)

with $\omega_{1s} = 6\pi$ rad/s.
Based on the internal model (4.43) of the unified disturbance signal $\xi(k)$, its state-space model representation is given by,

$$
\begin{bmatrix}
\dot{\tilde{z}}_1(k+1) \\
\dot{\tilde{z}}_2(k+1) \\
\dot{\tilde{z}}_3(k+1)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & -\alpha_s & \alpha_s
\end{bmatrix}
\begin{bmatrix}
\tilde{z}_1(k) \\
\tilde{z}_2(k) \\
\tilde{z}_3(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\varphi(k)
$$

(4.45)

where $\alpha_s = (2\cos(\omega_0\Delta_0) + 1)$, $z_1(k) = \xi(k)$, $z_3(k) = \xi(k+1)$, $z_3(k) = \xi(k+2)$, $A^{tilt}_{s\xi} \in \mathbb{R}^{3 \times 3}$, $B^{tilt}_{s\xi} \in \mathbb{R}^{3 \times 1}$, and $C^{tilt}_{s\xi} \in \mathbb{R}^{1 \times 3}$. Now, the disturbance model (4.45) is employed to define an extended state model of the spatial system (4.42); thus

$$
\begin{bmatrix}
\dot{x}(k+1) \\
\dot{z}(k+1)
\end{bmatrix} =
\begin{bmatrix}
A^{tilt}_{s\xi} & B^{tilt}_{s\xi} C^{tilt}_{s\xi} \\
0 & A^{tilt}_{s\xi}
\end{bmatrix}
\begin{bmatrix}
\dot{x}(k) \\
\dot{z}(k)
\end{bmatrix} +
\begin{bmatrix}
B^{tilt}_{s\xi} \\
B^{tilt}_{s\xi}
\end{bmatrix}
\psi(k)
$$

(4.46)

$$
M_{tilt}(k) =
\begin{bmatrix}
C^{tilt}_{s\xi} & 0 \\
C^{tilt}_{s\xi}
\end{bmatrix}
\begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix}
$$

where $\chi(k) = [x_1(k) \ x_2(k) \ x_3(k)]^T$, $\chi(k) = [z_1(k) \ z_2(k) \ z_3(k)]^T$, $A^{tilt}_{s\xi} \in \mathbb{R}^{5 \times 5}$, $B^{tilt}_{s\xi}, B^{tilt}_{s\xi} \in \mathbb{R}^{5 \times 1}$ and $C^{tilt}_{s\xi} \in \mathbb{R}^{1 \times 5}$.

Then, based on (4.42), (4.43), and (4.46), the following ADR observer-based control scheme is proposed.

**Theorem 4.3 (Unified disturbance $\xi(k)$ observer)** The estimation of the disturbance function $\tilde{\xi}(k)$, denoted as $\hat{\xi}(k)$, is given by the following ADR observer:

$$
\begin{align*}
\hat{\xi}_o(k+1) &= A^{tilt}_{s\xi} \hat{\xi}_o(k) + B^{tilt}_{s\xi} \psi(k) + L^{tilt}_\omega \left( M_{tilt}(k) - C^{tilt}_{s\xi} \hat{\xi}_o(k) \right) \\
\hat{\xi}(k) &= C^{tilt}_{s\xi a} \hat{\xi}_o(k)
\end{align*}
$$

(4.47)
where \( C_{s\xi}^{tilt} = \begin{bmatrix} 0 & C_{s\xi}^{tilt} \end{bmatrix} \), \( \hat{\xi}_o(k) = \begin{bmatrix} \hat{x}(k) & \hat{\xi}(k) \end{bmatrix}^T \) is the estimated state vector and \( L_{s}^{tilt} \) is the observer gain vector. Let \( L_{s}^{tilt} \), be chosen such that the eigenvalues of the matrix \( [A_{s\xi}^{tilt} - L_{s}^{tilt} C_{s\xi}^{tilt}] \) are into the unitary circle of the complex plane \( \mathbb{C} \). Then, the trajectories of the estimation error vector, \( \tilde{\xi}_z(k) = \xi_o(k) - \hat{\xi}_o(k) \), globally converge toward a small vicinity of zero where they remain ultimately bounded.

**Proof.** See Appendix A.7.

### Spatial ADR control law

The controller is designed to set, in a dominantly way, the characteristic polynomial of the tracking error. To do so, the control law is composed by:

- A disturbance rejection term, which is basically the disturbance estimation provided by the observer in Theorem 4.3. This term is used to cancel out the system nonlinearities and uncertainties, but also providing the rejection of disturbances at 0P and 3P frequencies.

- A linear feedback term, which contains a simple controller in charge of stabilizing the control loop.

The following theorem presents the proposed spatial ADR control scheme and Fig. 4.14 shows the complete observer/controller structure. The light blue blocks belong to the proposed control scheme in spatial domain.

**Theorem 4.4 (Spatial ADR Control Law)** Consider, in accordance with assumptions A4.1-A4.3 and regarding system (4.38), the following control law:

\[
\nu(k) = \frac{1}{\Delta_0^2} \begin{bmatrix} k_1 q + k_0^2 & q + k_2^2 \end{bmatrix} M_{tilt}(k) - \Delta_0^2 \hat{\xi}_z(k)
\]

where \( \hat{\xi}_z(k) \) is the estimation of the unified disturbance term.

The control law (4.48) makes the system dynamics be dominated by the polynomial

\[
p_M(z) = z^3 + (k_2^2 - 2) z^2 + (1 - 2k_2^2 - k_1^2) z + (k_2^2 - k_0^2),
\]

and with a proper selection of \( k_0 \), \( k_1 \) and \( k_2 \), such that polynomial \( p_M(z) \) has its roots into the unitary circle, the control law (4.48) takes \( M_{tilt}(k) \) to a vicinity of 0, rejecting the periodic components 0P and 3P of the fixed frame of reference of the wind turbine.

**Proof.** See Appendix A.8.
4.4.4 Results

The proposed ADR IPC schemes, *ADR Observer-based Control Scheme* (section 4.4.2) and *Spatial ADR Observer-based Control Scheme* (section 4.4.3), are validated using the FAST code [43] with a 5.0 MW reference nonlinear large-scale wind turbine [78]. Each control scheme is tested under realistic 3-D wind speed field generated in TurbSim [148] using Kaimal turbulence spectra with a mean value of 17 m/s at the hub height. The performance of each proposed IPC scheme is compared with two baseline control schemes: (1) GS/PI CPC without IPC action (see Sec. 4.3.4) named (CPC), and (2) GS/PI CPC with IPC1P action (see Fig. 4.15) named (CPC+MBC1P). Thus, in order to make a suitable comparison, all controllers are compared using the same GS/PI CPC scheme but with different IPC schemes.

The CPC scheme is a gain scheduling (GS) PI controller described in [78] to operate the 5 MW wind turbine in full load region. The baseline (CPC+MBC1P) is a conventional IPC scheme with 1P Coleman transform loop and an I controller \( G_c(s) = \frac{K_{i0}}{s} \) with \( K_{i0} = -6.5257 \times 10^{-6} \). This baseline can be observed in Fig. 4.15. The parameters of the system
model (4.26) were identified as follows: \( a_0 = b_0 = 1.2663 \) and \( a_1 = 3.9349 \). The spatial ADR/IPC scheme was designed with the following parameters: \( \Delta_0 = 0.005 \), \( \kappa = 1.2663 \), \( \omega_{1s} = 6\pi \), \( \alpha_s = 2.9911 \), \( L_s = [0.4621, 14.5095, 106.9282, 92.5182, 77.4836]^T \), \( k^s_0 = 0.0183 \), \( k^s_1 = -0.0188 \), \( k^s_2 = -0.76 \).

In order to evaluate the performance in load reduction, the results of the blade-root flap-wise bending moment \( M_y \), the hub-tilt moment \( M_{\text{tilt}}(t) \) and the hub-yaw moment \( M_{\text{yaw}}(t) \) are calculated in the form of the standard deviation which is an index of fatigue damage accumulation. Likewise, the standard deviation of the generated power under different wind profiles is also calculated. The results on load reduction and generated power of the baseline controllers (CPC) and (CPC+MBC1P), against the proposed ADR schemes: GS/PID CPC with ADR Individual Pitch Observer-Based Control named (CPC+MBC1P+ResObs) and GS/PID CPC with Spatial ADR Individual Pitch Observer-based control named (CPC+MBC1P+Spatial), are summarized in Tables 4.6, 4.7, 4.8 and 4.9.

Table 4.6 Standard deviation of flap-wise bending moments (kNm) for proposed control schemes under different wind profiles.

<table>
<thead>
<tr>
<th>Wind Profile</th>
<th>CPC</th>
<th>CPC+MBC1P</th>
<th>CPC+MBC1P+ResObs</th>
<th>CPC+MBC1P+Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% Turbulence</td>
<td>742.9</td>
<td>401.88 (54.1%)</td>
<td>386.87 (52.1%)</td>
<td>370.06 (49.8%)</td>
</tr>
<tr>
<td>10% Turbulence</td>
<td>1090.4</td>
<td>801.01 (73.5%)</td>
<td>776.21 (71.2%)</td>
<td>743.77 (68.2%)</td>
</tr>
<tr>
<td>25% Turbulence</td>
<td>1664.9</td>
<td>1381.5 (82.9%)</td>
<td>1341.9 (80.5%)</td>
<td>1291.7 (77.5%)</td>
</tr>
</tbody>
</table>
Table 4.7 Standard deviation of hub-tilt bending moments (kNm) for proposed control schemes under different wind profiles.

<table>
<thead>
<tr>
<th>Wind Profile</th>
<th>CPC</th>
<th>CPC+MBC1P</th>
<th>CPC+MBC1P+ResObs</th>
<th>CPC+MBC1P+Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% Turbulence</td>
<td>337.02</td>
<td>243.53(72.2%)</td>
<td>231.77(68.7%)</td>
<td>215.52(63.9%)</td>
</tr>
<tr>
<td>10% Turbulence</td>
<td>666.63</td>
<td>478.34(71.7%)</td>
<td>464.10(69.6%)</td>
<td>433.83(65.0%)</td>
</tr>
<tr>
<td>25% Turbulence</td>
<td>1130.3</td>
<td>820.98(72.6%)</td>
<td>798.30(70.6%)</td>
<td>754.08(66.7%)</td>
</tr>
</tbody>
</table>

Table 4.8 Standard deviation of hub-yaw bending moments (kNm) for proposed control schemes under different wind profiles.

<table>
<thead>
<tr>
<th>Wind Profile</th>
<th>CPC</th>
<th>CPC+MBC1P</th>
<th>CPC+MBC1P+ResObs</th>
<th>CPC+MBC1P+Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% Turbulence</td>
<td>345.74</td>
<td>275.96(79.8%)</td>
<td>245.36(70.9%)</td>
<td>227.48(65.7%)</td>
</tr>
<tr>
<td>10% Turbulence</td>
<td>685.74</td>
<td>544.84(79.4%)</td>
<td>491.09(71.6%)</td>
<td>456.48(66.5%)</td>
</tr>
<tr>
<td>25% Turbulence</td>
<td>1157.7</td>
<td>932.54(80.5%)</td>
<td>845.13(73.0%)</td>
<td>794.09(68.5%)</td>
</tr>
</tbody>
</table>

Table 4.9 Standard deviation of generated power (kW) for each control scheme under different wind profiles.

<table>
<thead>
<tr>
<th>Wind Profile</th>
<th>CPC</th>
<th>CPC+MBC1P</th>
<th>CPC+MBC1P+ResObs</th>
<th>CPC+MBC1P+Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% Turbulence</td>
<td>46.456</td>
<td>46.267</td>
<td>46.202</td>
<td>46.203</td>
</tr>
<tr>
<td>10% Turbulence</td>
<td>92.541</td>
<td>92.364</td>
<td>92.372</td>
<td>92.487</td>
</tr>
<tr>
<td>25% Turbulence</td>
<td>160.38</td>
<td>159.68</td>
<td>159.92</td>
<td>160.25</td>
</tr>
</tbody>
</table>

All proposed ADRC schemes provide reductions in the standard deviation of the loads $M_{y_123}(t)$, $M_{tilt}(t)$ and $M_{yaw}(t)$ in all wind profile cases no matter turbulence intensity (see Tables 4.6, 4.7 and 4.8). The proposed schemes render better load reduction of flap-wise, hub-tilt and hub-yaw bending moments compared to the baseline control schemes. The percentage reductions of flap-wise bending moments are in the range $[2.2\%, 2.6\%]$ and $[4.3\%, 5.8\%]$; the percentage reductions of hub-tilt bending moments are in the range $[2.2\%, 3.9\%]$ and $[6.1\%, 8.2\%]$; and the percentage reductions of hub-yaw bending moments are in the range $[7.6\%, 9.9\%]$ and $[11.4\%, 13.7\%]$, with the proposed ADR/IPC schemes (CPC+MBC1P+ResObs) and (CPC+MBC1P+Spatial), respectively. In all turbulent profiles, there are significant variations in the rotational speed of the WT that affect the performance of the proposed scheme (CPC+MBC1P+ResObs), because of the periodic components of blade-moments change of fundamental frequency as turbine speed changes. However, better results are shown when using the proposed ADR spatial scheme (CPC+MBC1P+Spatial), because the periodic components are invariant in spatial-domain. Note that, as seen in Table 4.9, the generated power is not affected by any of the IPC schemes.
The Fig. 4.16 shows the spectral content of the blade-root flap-wise bending moment for each closed-loop control scheme under different turbulent wind profiles. The dominance of the 1P frequency makes it easy to see that the 1P bending moment has been attenuated by all controllers except by the CPC, as expected. On the other hand, the reduction of the 2P and 4P frequencies can also be detailed in figs 4.16a, 4.16b and 4.16c for different wind turbulence intensities showing that (CPC+MBC1P+ResObs) and (CPC+MBC1P+Spatial) achieve better results than the baseline control schemes. This can be also verified in Fig. 4.17, where spectral content of $M_{tilt}(t)$ and $M_{yaw}(t)$ are shown for different turbulence intensities as well. In Figs 4.17a, 4.17b and 4.17c can be observed that the 3P component of $M_{tilt}(t)$ and $M_{yaw}(t)$ is attenuated by the proposed schemes under wind turbulence intensities from 5% to 25%. In particular, a better attenuation is shown by means of the ADR spatial scheme (CPC+MBC1P+Spatial).

It is important to point out that the proposed ADR/IPC schemes provide better results by means of: (a) using low-order models, (b) simplicity in the design inherited from ADR philosophy, and (c) just of one Coleman feedback loop.

4.5 Conclusions

4.5.1 Robust ADR collective pitch control scheme

In this chapter, an LMI-based robust active disturbance rejection collective pitch control scheme was proposed to solve the problem of speed and electric power regulation of a large variable-pitch wind turbine operating in full-load region and affected by high turbulence intensity. The proposed scheme is based on a robust extended state observer formulated within the Active Disturbance Rejection paradigm and tuned using pole-placement (LMI constraints) and LQ optimization.

In the proposed design, as well as in other ADRC approaches, model uncertainties and external disturbances are included in a general lumped disturbance input which is on-line estimated and subsequently rejected via the control law. However, the proposed design shows a methodology to allow ADR approaches handle LPV models and build observers that can both estimate disturbances and consider uncertainties or parameter variations from the design stage.

The simulations were developed using the FAST code in a 5.0MW reference nonlinear large-scale wind turbine with 3D realistic full field wind profiles. The results showed that the proposed ADR CPC scheme is robust, effective and provides better results in speed and power regulation than the industry standard gain-scheduling PID control.
Fig. 4.16 Closed loop simulation results of flap-wise bending moments for the control schemes under different turbulence wind profiles.
4.5 Conclusions

Fig. 4.17 Closed loop simulation results of tilt and yaw bending moments for the control schemes under different turbulence wind profiles.
4.5.2 ADR individual pitch control for periodic load reduction

In this chapter, two new control approaches to effectively address periodic load reduction in wind turbines operating in full load region are proposed. The proposed approaches tackle the load reduction problem based on the ADR philosophy by means of two simple but powerful observer-based schemes: an ADR/IPC resonant observer-based control technique and an ADR/IPC spatial-domain resonant observer-based control technique. In each proposed control technique, the observer was built with a suitable internal model (in the fixed frame of reference) that provides adequate disturbance estimations by means of: (a) high gain at a certain frequency point (3P) to assure accurate estimation of load components 2P and 4P in the flap-wise bending moments (rotating frame of reference), (b) high gain at zero frequency in order to eliminate the steady-state error in the fixed frame of reference 0P (attenuate 1P frequency in the rotating frame of reference), and (c) adequate tuning to estimate non-periodic loads and nonlinearities of the system over a range of frequencies from 0P to 4P.

High-fidelity wind-turbine simulation was conducted using the FAST code with a 5 MW reference wind turbine under 3D full field turbulent wind profiles, comparing the two new ADR control schemes against two benchmark schemes: GS/PID CPC and GS/PID CPC+MBC1P. The results of several realistic simulations on the FAST code showed that the new ADR approaches are effective in terms of better load reduction and power regulation with design simplicity.
Chapter 5

Concluding remarks

This thesis addressed three open problems of horizontal axis wind turbine control under the active disturbance rejection control framework. The energy capture maximization problem of wind turbines was addressed in chapter 3 by means of two new ADR solutions providing highlighted results. The first ADR solution was proposed by means of a GPI Control scheme whose main result is already published in [163]. The second solution was addressed by means of a dual GPI observer-based control scheme which is already published in [49].

The problem of speed/power regulation of wind turbines in full-load region was addressed in the first part of chapter 4 with a robust ADR observer-based control scheme. The proposed solution employed pole-placement and optimization tools to obtain the controller design parameters in order to handle the uncertain model of the wind turbine. The proposed approach showed better speed regulation and disturbance rejection than a gain scheduling PID baseline control scheme.

Finally, the problem of reduction of periodic disturbances on the rotor blades was addressed in the second part of chapter 4 by means of two new ADR observer-based control schemes. The first scheme tackles the problem using a time-domain resonant observer-based control scheme in order to attenuate the 1P, 2P and 4P frequency components of each rotor blade flap-wise bending moments. The second scheme addresses the problem by means of a new ADR spatial-domain resonant observer-based control strategy which was able to attenuate the 1P, 2P and 4P frequency components of each blade flap-wise bending moments. The proposed technique defined in the spatial-domain showed better disturbance reduction results than other known techniques. This proposed control scheme was submitted and is under review in [164]. Also, the problem of rejection of periodic disturbances for rotational mechatronic systems was addressed and the results were published in [142, 143].
References


[14] M. J. Balas and Y. J. Lee, “Stable disturbance accommodating control of large-
scale systems using singular perturbations with application to variable speed wind
turbines,” in Proceedings of the 30th Conference on Information Sciences and Systems,
(Princeton, NJ), 1996.

application to horizontal axis wind turbines,” in Proceedings of the 1998 ASME Wind

[16] K. Stol and M. Balas, “Full-state feedback control of a variable-speed wind turbine:
A comparison of periodic and constant gains,” Journal of Solar Energy Engineering-

wind turbines,” Journal of solar energy engineering, vol. 125, no. 4, pp. 396–401,
2003.


part 2 performance of control system,” in European Wind Energy Conference, vol. 1,


wind turbines: standard and adaptive techniques for maximizing energy capture,”

[23] S. Suryanarayanan and A. Dixit, “Control of large wind turbines: Review and sug-
gested approach to multivariable design,” in Proceedings of National Conference on
Control and Dynamic Systems, (Mumbai, India), 2005.


[26] P. W. Carlin, A. S. Laxson, and E. B. Muljadi, “The history and state of the art of
variable-speed wind turbine technology,” Wind Energy, vol. 6, no. 2, pp. 129–159,
2003.

[27] K. Z. Ostergaard, Robust, Gain-Scheduled Control of Wind Turbines. PhD thesis,
Aalborg University, 2008.


References


Appendix A

Proofs

A.1 Proof of Theorem 3.1

By subtracting the proposed observer (3.17) from the augmented system state equation (3.15), the following estimation error dynamics is obtained:

\[ \dot{\tilde{e}}_x(t) = (A - L^\tau C)\tilde{e}_x(t) + B_a T_r^{(p)}(t) = A_{e_x}\tilde{e}_x(t) + B_a T_r^{(p)}(t) \quad (A.1) \]

where the eigenvalues of \( A_{e_x} = (A - L^\tau C) \) can be placed as desired by selecting the gain vector \( L^\tau \).

In order to obtain an ultimate bound for \( \tilde{e}_x(t) \), let \( Q \in \mathbb{R}^{(p+3) \times (p+3)} \) be a constant, positive definite symmetric matrix. The proper stable character of the matrix \( A_{e_x} \) implies the existence of a positive definite matrix \( P \in \mathbb{R}^{(p+3) \times (p+3)} \) such that \( A_{e_x}^T P + PA_{e_x} = -Q \). Consider the Lyapunov function candidate \( V(\tilde{e}_x(t)) = \frac{1}{2}\tilde{e}_x^T(t)P\tilde{e}_x(t) \). The time derivative of \( V(\tilde{e}_x(t)) \) satisfies

\[ \dot{V}(\tilde{e}_x,t) = \frac{1}{2} \left[ \tilde{e}_x^T(t)(-Q)\tilde{e}_x(t) \right] + B_a^T P\tilde{e}_x(t)T_r^{(p)}(t). \quad (A.2) \]

For \( Q = I \), that is, an identity matrix, \( \dot{V}(\tilde{e}_x,t) \) satisfies

\[ \dot{V}(\tilde{e}_x,t) = \frac{1}{2} \left[ \tilde{e}_x^T(t)(-Q)\tilde{e}_x(t) \right] + B_a^T P\tilde{e}_x(t)T_r^{(p)}(t) \leq -\frac{1}{2} \|\tilde{e}_x(t)\|^2 + \|B_a\|^2 \|P\|^2 \|\tilde{e}_x(t)\|_2 K_{T_r} < 0 \quad (A.4) \]

Given that \( \|B_a\|^2 = 1 \) and according to (A.4), \( \dot{V}(\tilde{e}_x,t) \) is strictly negative if

\[ \|\tilde{e}_x(t)\|_2 > 2K_{T_r}\|P\|_2 \quad (A.5) \]
Therefore, \( \dot{V}(\tilde{e}_x, t) \) is strictly negative outside the following disc:

\[
D_x = \{ \tilde{e}_x(t) \in \mathbb{R}^{p+3}, \| \tilde{e}_x(t) \|_2 \leq 2K_T\|P\|_2 \}. \tag{A.6}
\]

Consequently, a uniform ultimate bounded (UUB) result was obtained regarding the estimation error variables \( \tilde{e}_x(t) \).

### A.2 Proof of Theorem 3.2

Let us define the tracking error in the complex variable \( s \) as \( E_{\omega_r}(s) = \omega_r(s) - \omega_{\text{opt}}(s) \). From (3.21), (3.23) and (3.31) the tracking error dynamics is given by:

\[
p_{\omega_r}(s)E_{\omega_r}(s) = s^{m+1} (s^3 + k_{m+7}s^2 + k_{m+6}s + k_{m+5}) \xi(s). \quad (A.7)
\]

Then, a transfer function relating the tracking error \( E_{\omega_r}(s) \) and the unified disturbance function \( \xi(s) \) is derived and then decomposed in partial fractions, thus

\[
G_{\epsilon\xi}(s) = \frac{E_{\omega_r}(s)}{\xi(s)} = \frac{s^{m+1} (s^3 + k_{m+7}s^2 + k_{m+6}s + k_{m+5})}{p(s)} = \frac{\alpha_1}{s - \sigma_1} + \frac{\alpha_2}{s - \sigma_2} + \ldots + \frac{\alpha_{m+8}}{s - \sigma_{m+8}} \quad (A.8)
\]

with \( \alpha_1, \alpha_1, \ldots, \alpha_{m+8} \in \mathbb{C} \) and \( \sigma_1, \sigma_1, \ldots, \sigma_{m+8} \in \mathbb{C} \) their corresponding stable poles. Then, an ultimate bound for the tracking error can be obtained as:

\[
\lim_{t \to \infty} \sup |e_{\omega_r}(t)| < k_\xi \left[ \left| \frac{\alpha_1}{\text{Re} \{ \sigma_1 \}} \right| + \left| \frac{\alpha_2}{\text{Re} \{ \sigma_2 \}} \right| + \ldots + \left| \frac{\alpha_{m+8}}{\text{Re} \{ \sigma_{m+8} \}} \right| \right]. \quad (A.9)
\]

Therefore, the asymptotic convergence of the tracking error \( e_{\omega_r}(t) \) and its bounding disk ratio, can be arbitrary governed selecting the real part of the tracking error poles in \( p_{\omega_r}(s) \).

### A.3 Proof of Theorem 3.3

By subtracting the proposed GPI observer (3.42) from the augmented system state equation (3.41), the following estimation error dynamics is obtained

\[
\dot{\tilde{e}}_{xc}(t) = \left( A_c - L^{\Delta_1}C_c \right) \tilde{e}_{xc}(t) + B_{c3}\Delta_1^{(m)}(t) \\
= A_{\tilde{e}_{xc}} \tilde{e}_{xc}(t) + B_{c3}\Delta_1^{(m)}(t) \quad (A.10)
\]

\[= A_{\tilde{e}_{xc}} \tilde{e}_{xc}(t) + B_{c3}\Delta_1^{(m)}(t) \quad (A.11)\]
where the roots of $|sI - A_{e_{sc}}| = s^{m+1} + l_{m+1}^A s^{m} + \cdots + l_{2}^A s + l_{1}^A$ can be placed as desired by selecting the gain vector $L^A$.

Following the same idea of proof A.1, let $Q_e = I \in \mathbb{R}^{(m+1)\times(m+1)}$ be a constant, positive definite symmetric matrix; then, a positive definite matrix $P_e \in \mathbb{R}^{(m+1)\times(m+1)}$ exists, such that $A_{e_{sc}}^T P_e + P_e A_{e_{sc}} = -Q_e$. Consider the Lyapunov function candidate $V(\tilde{e}_{x_e}(t)) = \frac{1}{2}\tilde{e}_{x_e}^T(t)P_e\tilde{e}_{x_e}(t)$. The time derivative of $V(\tilde{e}_{x_e}(t))$, that is, $\dot{V}(\tilde{e}_{x_e},t)$ is strictly negative outside the disc:

$$D_{x_e} = \{ \tilde{e}_{x_e}(t) \in \mathbb{R}^{m+1}, \|\tilde{e}_{x_e}(t)\|_2 \leq 2K_{\Delta_{1}} \|P_e\|_2 \}.$$  \hspace{1cm} (A.12)

Consequently, a uniform ultimate bounded (UUB) result was obtained regarding the estimation error variables $\tilde{e}_{x_e}(t)$.

### A.4 Proof of Theorem 3.4

By replacing (3.45) in (3.37), the following dynamics is obtained:

$$\dot{\omega}_y(t) - \dot{\omega}_{\text{opt}}(t) + k_0^\phi (\omega_y(t) - \dot{\omega}_{\text{opt}}(t)) = -\hat{\phi}(t) - \dot{\Lambda}_{1}(t) + \Delta_{1}(t) + \varphi(t)$$ \hspace{1cm} (A.13)

Then, by defining some estimation errors: $\tilde{e}_{\omega_{\text{opt}}}(t) = \omega_{\text{opt}}(t) - \dot{\omega}_{\text{opt}}(t)$, $\tilde{e}_{\Delta_{1}}(t) = \Delta_{1}(t) - \dot{\Lambda}_{1}(t)$, and $\tilde{\varphi}(t) = \varphi(t) - \hat{\phi}(t)$ and replacing them into (A.13), the following control system tracking error dynamics is obtained:

$$\begin{align*}
\left( \dot{\omega}_y(t) - \dot{\omega}_{\text{opt}}(t) \right) + k_0^\phi (\omega_y(t) - \omega_{\text{opt}}(t)) &= -\dot{\omega}_{\text{opt}}(t) - k_0^\phi \tilde{e}_{\omega_{\text{opt}}}(t) + \tilde{e}_{\Delta_{1}}(t) + \tilde{\varphi}(t) \\
\dot{e}_y(t) + k_0^\phi \dot{e}_y(t) &= -\dot{e}_{\omega_{\text{opt}}}(t) - k_0^\phi \tilde{e}_{\omega_{\text{opt}}}(t) + \tilde{e}_{\Delta_{1}}(t) + \tilde{\varphi}(t)
\end{align*}$$  \hspace{1cm} (A.14)

Therefore, as long as $k_0^\phi > 0$ and the estimation errors $\dot{e}_{\omega_{\text{opt}}}(t)$, $\dot{e}_{\omega_{\text{opt}}}(t)$, $\tilde{e}_{\Delta_{1}}(t)$ and $\tilde{\varphi}(t)$ are ultimately bounded by the GPI observers (3.17) and (3.42), the tracking error dynamics $e_y(t)$ will remain stable and bounded since the right side of (A.15) is also bounded.
A.5 Proof of Theorem 4.1

By subtracting the observer (4.32) from the augmented system equation (4.31), the following estimation error dynamics is obtained:

\[
\dot{\tilde{e}}_{x_o} = A_a \tilde{e}_{x_o}^{\text{tilt}} - L_{\text{tilt}}^{\text{tilt}} \left(C_{a}^{\text{tilt}} x_o^{\text{tilt}} - C_{a}^{\text{tilt}} \dot{x}_o^{\text{tilt}} \right) + B_{da}^{\text{tilt}} \text{im}_{\text{tilt}}(t) \tag{A.16}
\]

\[
\dot{\tilde{e}}_{x_o} = A_a \tilde{e}_{x_o}^{\text{tilt}} - L_{\text{tilt}}^{\text{tilt}} C_{a}^{\text{tilt}} \tilde{e}_{x_o}^{\text{tilt}} + B_{da}^{\text{tilt}} \text{im}_{\text{tilt}}(t) \tag{A.17}
\]

\[
\dot{\tilde{e}}_{x_o} = \left(A_a^{\text{tilt}} - L_{\text{tilt}}^{\text{tilt}} C_{a}^{\text{tilt}} \right) \tilde{e}_{x_o}^{\text{tilt}} + B_{da}^{\text{tilt}} \text{im}_{\text{tilt}}(t) \tag{A.18}
\]

where the eigenvalues of $A_{e_{x_o}}$ can be placed to the left of the imaginary axis of the complex plane $s$ as desired, by selecting the gain vector $L_{\text{tilt}}^{\text{tilt}}$.

In order to obtain an ultimate bound of $\tilde{e}_{x_o}^{\text{tilt}}$, let $Q \in \mathbb{R}^{5\times5}$ be a constant, positive definite symmetric matrix. The proper stable character of the matrix $A_{e_{x_o}}$ implies the existence of a positive definite matrix $P \in \mathbb{R}^{5\times5}$ such that $A_{e_{x_o}}^T P + P A_{e_{x_o}} = -Q$. Consider the Lyapunov function candidate

\[
V(\tilde{e}_{x_o}^{\text{tilt}}(t)) = \frac{1}{2} \left[ \tilde{e}_{x_o}^{\text{tilt}}(t) \right]^T P \tilde{e}_{x_o}^{\text{tilt}}(t) + B_{da}^{\text{tilt}} P \tilde{e}_{x_o}^{\text{tilt}}(t) \text{im}_{\text{tilt}}(t). \tag{A.19}
\]

Then, assuming that the disturbance input $d_{\text{tilt}}(t)$—in the fixed frame of reference—is dominantly composed by $d_{\text{tilt}}(t) = d_0 + \sin(\omega_3 p t)$ with $d_0$ a finite constant, therefore $\text{im}_{\text{tilt}}(t) = \left( \tilde{d}_{\text{tilt}} + \omega_3^2 p \tilde{d}_{\text{tilt}} \right)$ exhibits an uniform and absolute bound. This condition assures the existence of an unknown but finite constant, $K_{d_{\text{tilt}}}$, such that

\[
\sup_{t \geq 0} |\text{im}_{\text{tilt}}(t)| \leq K_{d_{\text{tilt}}}.
\]

For $Q = I$, that is, an identity matrix, $V(\tilde{e}_{x_o}^{\text{tilt}}(t))$ satisfies

\[
V(\tilde{e}_{x_o}^{\text{tilt}}(t)) \leq -\frac{1}{2} \left\| \tilde{e}_{x_o}^{\text{tilt}}(t) \right\|^2_2 + ||P||_2 \left\| \tilde{e}_{x_o}^{\text{tilt}}(t) \right\|_2 K_{d_{\text{tilt}}} < 0. \tag{A.20}
\]

Given that $||B_{da}^{\text{tilt}}||_2 = 1$ and according to (A.20), $V(\tilde{e}_{x_o}^{\text{tilt}}(t))$ is strictly negative if

\[
\left\| \tilde{e}_{x_o}^{\text{tilt}}(t) \right\|_2 > 2K_{d_{\text{tilt}}} ||P||_2. \tag{A.21}
\]
Therefore, $\dot{V}(\tilde{e}_{xo}^{tilt}, t)$ is strictly negative outside the following disc:

$$D_{xo} = \left\{ e_{xo}^{tilt} (t) \in \mathbb{R}^5, \left\| e_{xo}^{tilt} (t) \right\|_2 \leq 2K_{dtilt} \| P \|_2 \right\}.$$  \hfill (A.22)

Consequently, the state estimation error vector $\tilde{e}_{xo}^{tilt} (t)$ converges towards the interior of the disk $D_{xo}$ centered in the origin where remains ultimately bounded.

### A.6 Proof of Theorem 4.2

Applying the control law (4.33) into the system (4.26), and after some algebraic manipulations, the closed-loop system dynamics results in:

$$\dot{e}_{xo}^{tilt} = \left( A_{xo}^{tilt} - B_{xo}^{tilt} K_{xo}^{tilt} \right) e_{xo}^{tilt} + B_{xo}^{tilt} K_{xo}^{tilt} \tilde{e}_{xo}^{tilt} + B_{xo}^{tilt} \tilde{e}_{d}^{tilt},$$  \hfill (A.23)

where $\tilde{e}_{xo}^{tilt} = x_{xo}^{tilt} - \hat{x}_{xo}^{tilt}$ is the state estimation error vector and $\tilde{e}_{d}^{tilt} = d_{xo}^{tilt} - \hat{d}_{xo}^{tilt}$ is the disturbance estimation error. The convergence of both, $\hat{d}_{xo}^{tilt}$ towards a vicinity of $d_{xo}^{tilt}$ and $\hat{x}_{xo}^{tilt}$ towards a vicinity of $x_{xo}^{tilt}$, under an appropriate configuration of the GPI/ADR observer (4.32), establishes that the term $B_{xo}^{tilt} K_{xo}^{tilt} \tilde{e}_{xo}^{tilt} + B_{xo}^{tilt} \tilde{e}_{d}^{tilt}$ of (A.23) evolves within a sufficiently small vicinity of zero in a uniformly ultimately bounded fashion (see proof of Theorem 4.1). As a result, the closed-loop system dynamics is strongly dominated by the eigenvalues of the matrix

$$\left( A_{xo}^{tilt} - B_{xo}^{tilt} K_{xo}^{tilt} \right).$$

With the convergence of $\tilde{e}_{xo}^{tilt}$ and $\tilde{e}_{d}^{tilt}$ toward a small vicinity of zero, and provided that the eigenvalues of $\left( A_{xo}^{tilt} - B_{xo}^{tilt} K_{xo}^{tilt} \right)$, under a suitable selection of $K_{xo}^{tilt}$, are located to the left of the imaginary axis of the complex plane $s$, the components 0P and 3P of the bending moment $M_{xo}^{tilt} (t)$ converge towards a small vicinity of zero.

### A.7 Proof of Theorem 4.3

The eigenvalues of $A_{\xi o} = [A_{xo}^{tilt} - L_{xo}^{tilt} C_{xo}^{tilt}]$ can be placed as desired by selecting the gain vector $L_{xo}^{tilt}$, such that the eigenvalues of $A_{\xi o}$ are located inside the unitary circle of the complex plane $\mathbb{C}$. The estimation error $e_{\xi}^{o} (k)$ is restricted to a vicinity of the origin of the estimation error phase space as spatial-clock elapses. The size of the vicinity is related to the
achieved size of attenuation in the term

\[
\sup_k \left| (q-1) \left( q^2 - 2 \cos(\omega_1 \Delta \theta) q + 1 \right) \tilde{\xi}(k) \right| \leq K_{\tilde{\xi}}.
\]

To study the ultimate boundedness, we consider the following Lyapunov function candidate \( V(\tilde{\xi}(k)) = \frac{1}{2} \left[ \tilde{\xi}(k) \right]^T P [\tilde{\xi}(k)] \), with \( P \) positive definite. Then, We need to find

\[
\Delta V(\tilde{\xi}(k)) = V(\tilde{\xi}(k+1)) - V(\tilde{\xi}(k)),
\]

and after some trivial manipulations, it is concluded that:

\[
\Delta V(\tilde{\xi}(k)) = \frac{1}{2} \left[ \tilde{\xi}(k) \right]^T \left[ A_{\tilde{\xi}_o}^T P A_{\tilde{\xi}_o} - P \right] [\tilde{\xi}(k)] + \frac{1}{2} \left[ \tilde{\xi}(k) \right]^T A_{\tilde{\xi}_o}^T P B_{\tilde{\xi}_o} \tilde{\imath}^{illt}(k)
\]

\[
+ \frac{1}{2} \left( \tilde{\imath}^{illt}(k) \right)^2 \left[ B_{\tilde{\xi}_o}^{illt} \right]^T P B_{\tilde{\xi}_o}^{illt}.
\]

The stable character of \( A_{\tilde{\xi}_o} \) implies that for every constant, symmetric positive definite \( 5 \times 5 \) matrix \( Q = Q^T \), there exists a symmetric positive definite \( 5 \times 5 \) matrix \( P = P^T \) such that:

\[
\left[ \tilde{\xi}(k) \right]^T \left[ A_{\tilde{\xi}_o}^T P A_{\tilde{\xi}_o} - P \right] [\tilde{\xi}(k)] < \left[ \tilde{\xi}(k) \right]^T [-Q] [\tilde{\xi}(k)] < 0
\]

Therefore, equation (A.25) results in:

\[
\Delta V(\tilde{\xi}(k)) \leq -\frac{1}{2} \left[ \tilde{\xi}(k) \right]^T Q [\tilde{\xi}(k)] + \frac{1}{2} \left[ \tilde{\xi}(k) \right]^T A_{\tilde{\xi}_o}^T P B_{\tilde{\xi}_o} \tilde{\imath}^{illt}(k)
\]

\[
+ \frac{1}{2} \left( \tilde{\imath}^{illt}(k) \right)^2 \left[ B_{\tilde{\xi}_o}^{illt} \right]^T P B_{\tilde{\xi}_o}^{illt}.
\]

For \( Q = I \), that is an \( (5) \times (5) \) identity matrix, and given that \( \left\| B_{\tilde{\xi}_o}^{illt} \right\|_2 = 1 \) and according to assumption A4.3 given in equation (4.44), this function satisfies:

\[
\Delta V(\tilde{\xi}(k)) \leq -\frac{1}{2} \left\| \tilde{\xi}(k) \right\|_2^2 + \left\| \tilde{\xi}(k) \right\|_2 A_{\tilde{\xi}_o}^T \left\| \tilde{\xi}(k) \right\|_2 \left\| P \right\|_2 + \frac{1}{2} K_{\tilde{\xi}} \left\| P \right\|_2.
\]

By solving the quadratic equation on \( \left\| \tilde{\xi}(k) \right\|_2 \), it is straight forward to obtain that \( \Delta V(\tilde{\xi}(k)) \) is strictly negative everywhere outside the sphere \( S_{\tilde{\xi}_o} \), given by:

\[
S_{\tilde{\xi}_o} = \left\{ \tilde{\xi}(k) \in \mathbb{R}^5, \left\| \tilde{\xi}(k) \right\|_2 \leq K_{\tilde{\xi}} \left[ \left\| A_{\tilde{\xi}_o}^T \right\|_2 \left\| P \right\|_2 + \sqrt{\left\| A_{\tilde{\xi}_o}^T \right\|_2^2 \left\| P \right\|_2^2 + \left\| P \right\|_2^2} \right] \right\}, \quad (A.29)
\]
hence, all trajectories \( \tilde{\xi} (k) \) starting outside of this sphere converge towards its interior, and all those trajectories starting inside \( S_{\tilde{\xi}} \) will never abandon it.

### A.8 Proof of Theorem 4.4

The closed-loop system dynamics can be found by substituting the control law (4.48) in the system (4.38), which after simple algebraic manipulation, results in:

\[
\left( (q + k_2^s) (q - 1)^2 - (k_1^s q + k_0^s) \right) M_{tilt} (k) = \Delta_0^2 (q + k_2^s) \left( \tilde{\xi} (k) - \hat{\xi} (k) \right).
\]

The convergence of \( \hat{\xi} (k) \) towards a small vicinity of \( \xi (k) \), under an appropriate design of the GPI observer, establishes that the right hand side of (A.30) evolves within a sufficiently small vicinity of zero in a uniformly ultimately bounded fashion (see proof of Theorem 4.3). As a result, the system dynamics is strongly dominated by the characteristic polynomial

\[
p_M (\tilde{\xi}) = \tilde{\xi}^3 + (k_2^s - 2) \tilde{\xi}^2 + (1 - 2k_2^s - k_1^s) \tilde{\xi} + (k_2^s - k_0^s).
\]

Finally, it can be concluded that with a convergence of \( \hat{\xi} (k) \) towards an arbitrarily small vicinity of \( \xi (k) \) according to the observer’s bandwidth (specially at 0P and 3P frequencies), and provided that the roots of \( p_M (\tilde{\xi}) \) are located into the unitary circle, the bending moment \( M_{tilt} (k) \) converges towards a small vicinity of zero at 0P and 3P frequencies, which are equivalent to the 1P, 2P and 4P frequencies in the rotating coordinate system of the wind turbine.