Applying TOC Heuristics to Job Scheduling in a Hybrid Flexible Flow Shop

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Abstract

This paper introduces an application of the Theory of Constraints product mix heuristic to job scheduling in a Hybrid Flexible Flow Shop. The general heuristic is adapted for unrelated parallel machines and the algorithm is implemented as a job detailed scheduling tool based on the principle of the Theory of Constraints to schedule the production based in the bottleneck resource. The adaptation of the methodology to a flexible hybrid context, where there is parallelism in the bottleneck stage, and its application in a textile plant, helps to assign capacity based on the contribution margin. The result is a viable job scheduling focused on the profitability unit. Although the results do not reach the global optimum of this type of problems, they represent a fast and effective job scheduling alternative in the contexts under study.

Keywords: Theory of Constraints; Flow Shop; job scheduling; heuristics

1. Introduction

Production scheduling is the stage that defines the assigning of specific jobs on a machine or set of machines and the sequence or order in which the pending jobs will be processed. Efficient production programs can lead to substantial improvements in productivity and cost reduction. [1]

Each production environment has its own restrictions and particularities that require the application of appropriate techniques to ensure efficient scheduling.

The hybrid Flow Shop is a production line process in which at least one of the stages includes parallel machines. In a flexible Hybrid Flow Shop, some products might be processed without going through one or more of the stages. [2] A classic example of such a process is in the textile industry.

The Scheduling in the Hybrid Flow Shop has been approached by dozens of researchers using different techniques. Some of the more recent contributions are those of Qiao and Sun, [3](2011), Yue-Wen et al. [4](2011) and Yalaoui et al. [5](2011) who applied intelligent particles, and Hidri and Haouari [6] (2011) who applied limitation strategies.

One of the main features in the flexible hybrid Flow Shop is parallelism in at least one stage of the production process. The most general and most common case in the
real world [7][8] is that of unrelated parallel machines. These are, machines with different production rates and different job scheduling possibilities. Some recent work on parallel machines scheduling includes: Zhank and Van de Velde [9](2012) who proposed an approximation algorithm, Driessel and Monch [10](2011) and James and Almada-Lobo [11](2011) with Variable neighborhood search, Lin et al. [12](2011) who applied a greedy algorithm, while Chang and Chen [13](2011) and Arango et al. [14](2013) adapted genetic algorithms.

It is common to find the use of heuristics. In the most general sense heuristics refer to those smart technical methods or procedures required to perform a task. Heuristics is the result of the knowledge of an expert and does not come from a rigorous formal analysis.

The optimal "product-mix" of the Theory of Constraints is obtained using a heuristic known as TOC, which was developed based on the five steps proposed by Goldratt [16] (1984). (Find the constraint, exploit the constraint, subordinate the system to the constraint, elevate the constraint and when the constraint is overcome, find a new one and repeat the process).[16]

Several researchers have taken into consideration this algorithm: (Fredendall and Lea [17] 1997; Lee and Plenert [18], 1993), who discussed the capacity of TOC compared to LP or ILP models (Lea and Fredendall [19] 2002; Mabin and Davies [20], 2003; Aryanethad and Komijan [21], 2004; Souren, Ahn and Schmitz [22](2005).

Metaheuristics have also been applied to the problem of the optimal mix under study using the Theory of Constraints: Onwubolu [23](2001) proposed an algorithm based on tabu search; Mishra, Prakash, Tiwari, Shankar, and Chan [24] (2005) presented a hybrid algorithm of tabu search and simulated annealing; and Onwubolu and Mutingi [25] (2001) developed a genetic algorithm.

Step 1: Identify the system constraints: Calculate the required load on each resource to manufacture all the products. The constraint or bottleneck (BN) is the resource in which demand exceeds capacity.

Step 2: Decide how to exploit the system constraints: (a) Calculate the contribution margin (CM) of each product such as the selling price minus costs of raw materials (RM).

(b) Calculate the relation between the contribution margin of the products and the processing time at the bottleneck resource (CM/BN).

(c) Reserve the capacity of the bottleneck resource, by sorting the products in descending order according to the relation CM/BN.

Industrial processes classified as "Hybrid Flexible Flow Shop", such as in the case of textile production, count on several similar resources in parallel with similar process routes for all the products regarding the order in which these go through the transformation processes. The issue is to find out how to distribute each job in the different resources with the aim to favor contribution to the business profits.

Figure 1. DEA analysis (Data Envelopment Analysis) Textile Process. Source: Authors´ contribution
The figure shows that the process of looms, with an indicator of 0.754, is the least efficient of all and the furthest away from the efficiency frontier. Therefore, it is determined that the process of looms is the bottleneck of the system.

**Step 2: Decide how to exploit the system constraints:**

Calculate the contribution margin (CM) of each product, the selling price minus costs of raw materials (RM).

The throughput (contribution margin), as stated by the procedure, ignores the processing costs which can have, in many cases, a high impact on total cost. The difference in profitability of different products, depending on technical characteristics and the required processes, can also be established.

Therefore, it is proposed to calculate the contribution margin as the difference between the sales price and manufacturing costs with the objective to arrive at a model. This model might be considered when making scheduling and production decisions at the operational and strategic levels, and might also be used to obtain results comparable with the determination of the optimal product mix using the traditional methodology of operations research focused on the strategic level.

In this way:

\[ CM_i = SP_i - UC_i \]  

(1)

Where:

- \( CM_i \) = Contribution margin per kilogram of product \( i \).
- \( SP_i \) = Selling price per kilogram of product \( i \).
- \( UC_i \) = Manufacturing Unit Cost per kilogram of product \( i \).

Calculate the relation between the contribution margins of the products per processing time at the bottleneck resource (CM/BN).

The formula below might be used to get the contribution margin per unit of time in the bottleneck resource:

\[ (CM/BN)_i = \frac{(SP_i - UC_i)gm_{i}rpm_{i}el_{i}lo_{i}}{100000dw_{i}} \]  

(2)

Where:

- \( (CM/BN)_i \) = Contribution margin of product \( i \) per minute on loom \( i \).
- \( gm_{i} \) = Weight in grams per meter of product \( i \).
- \( rpm_{i} \) = Average speed in revolutions per minute of the looms on which product \( i \) is processed.
- \( el_{i} \) = Efficiency in the loom of product \( i \).
- \( lo_{i} \) = Loom simultaneous outputs when product \( i \) is processed.
- \( dw_{i} \) = Density in wefts/centimeters in product \( i \).

The constant 100000 corresponds to the conversion of units: 100 match centimeters of \( dw_{i} \) to the meters of \( gm_{i} \) and 1000 for the consistency between grams of \( gm_{i} \) and kilograms of \( (SP_i - UC_i) \).

The capacity of the bottleneck resource must be reserved sorting the products in descending order according to the relation \( CM/BN \), until capacity is depleted.

In this step, a ranking of profitability per unit of time of the different products must be established until the capacity of the bottleneck resource is depleted, in descending order, starting with the product with the highest contribution margin.

The problem is compounded when, as happens in textile weaving, there are multiple parallel machines with different capacities, and product differentiation depends on the technical specifications of the loom on which textiles are weaved. It must then be considered that the parallel machines as unrelated.

The decision, therefore, is not only related to the distribution of productive capacity among a set of jobs waiting to be processed on a resource. An optimal distribution of work is required, that meets the criterion of maximizing total contribution margin, and the scheduling of resources according to technical features taking into account the limitations of the market and the availability of raw materials.

The optimization model is thus formulated:

\[ \text{Maximize } z = \sum_{i=1}^{n} \sum_{l=1}^{s} (CM/BN)_i y_{il} \]  

(3)

Subject to:

**Capacity constraints:**

Assuming a month of thirty working days, 24 hours a day, 60 minutes per hour:

\[ \sum_{i=1}^{n} y_{il} \leq 60 \times 24 \times 30, \quad l = 1,2,...,s \]  

(4)

**Market constraints:**

\[ \sum_{i=1}^{n} \sum_{l=1}^{s} \frac{gm_{i}rpm_{i}el_{i}lo_{i}}{100000dw_{i}sr_{i}^{l}} y_{il} \leq U_{i} \]  

(5)

**Raw material constraints:**

\[ \sum_{i=1}^{n} \sum_{l=1}^{s} \frac{gm_{i}rpm_{i}el_{i}lo_{i}}{100000dw_{i}sr_{i}^{l}} y_{il} \leq R_{k} \]  

(6)

**Non-negativity condition:**

\[ y_{il} \geq 0, \quad i = 1,2,..., n \quad l = 1,2,..., s \]  

(7)

Where:

- \( z \) = Total contribution margin
- \( (CM/BN)_i \) = Contribution margin of product \( i \) per minute of work in the loom.
- \( y_{il} \) = Minutes for product \( i \) on loom \( l \).
\[ i = \text{Product}. \]
\[ n = \text{Total amount of products}. \]
\[ d_{wi} = \text{Density in wefts/centimeters of product } i. \]
\[ sr_i = \text{Size of roll of product } i (\text{kilograms}). \]
\[ gm_i = \text{Linear weight of product } i (\text{grams/meters}). \]
\[ el_i = \text{Efficiency in looms of product } i \text{ (ratio of 0 (not efficient) and 1 (fully efficient)).} \]
\[ lo_i = \text{Number of loom outputs used simultaneously on a loom to make product } i. \]
\[ rpm_l = \text{Loom speed } l (\text{revolutions per minute}). \]
\[ l = \text{Loom}. \]
\[ s = \text{Total number of looms}. \]
\[ U_i = \text{Maximum demand of product } i. \text{ (rolls).} \]
\[ r_k = \text{Demand of raw material } k \text{ in kilometers per roll of product } i. \]
\[ R_k = \text{Available kilograms of raw material } k. \]
\[ q = \text{Total amount of raw material}. \]

This model, according to the initial philosophy of TOC heuristics, must be resolved by sorting products from the highest to the lowest contribution margin. The proposed algorithm includes a flexibility indicator of the loom as a criterion for job assignation. The total of minutes required by each process is distributed among the looms to complete the maximum rolls of each product, taking into account technical constraints:

**Step 1:** Sort products from the highest to the lowest value of CM / BN.

**Step 2:** Calculate the flexibility indicator in each loom. This is calculated as the result of dividing the number of groups in which the loom is scheduled by the lower number of looms among all the groups where the loom belongs. For example, the flexibility indicator of a loom that belongs to a group of 6 looms and a group of 3, will be 2/3. And the flexibility indicator of another loom that belongs to two groups of 4 and 5 looms, will be 3/4.

**Step 3:** Sort in ascending order the looms according to the flexibility indicator calculated in step 2. In the above example the first loom is better sorted (indicator = 2/3) than the second loom (indicator = 3/4). This rule favors products that can be manufactured in very few looms and these looms should be scheduled only as needed. Similarly, the looms with the highest flexibility are assigned at the end thereby facilitating the scheduling of more products.

**Step 4:** Assign the first product of the current list to the first of the available looms with capacity to manufacture. The assignment corresponds to the lower value in rolls equal to the available capacity on the loom, the maximum demand of the unscheduled product, and the availability of raw material for this product. Then subtract the assigned value from the available capacity of the loom, from the maximum product demand and from the availability of raw materials.

\[ y_{il} = \min(cap_l, dem_i, rpm_k) \]  

(8)

Where:
\[ y_{il} = \text{Job assignment } i \text{ to loom } l \]
\[ cap_l = \text{Available capacity in loom } l \]
\[ dem_i = \text{Maximum demand of product } i \]
\[ rpm_k = \text{Raw materials } k \]

**Step 5:** When maximum demand is not covered with one loom, continue to the next loom. When maximum demand is covered, the product should be removed from the list and return to step 4. Continue until the list of products is exhausted.

At the end, a detailed product assignation of each of the productive resources must be obtained considering market constraints, raw materials and machine capacities. The algorithm is based on the original TOC heuristics and tries to respect its main premises. A general integer optimization algorithm might be applied based on the model presented in expressions (3) a (7) (such as the hybrid genetic simplex introduced in [28]).

Thus, it is necessary to depart from the standard procedure of the Theory of Constraints to achieve the optimal product mix, following strategies such as those by different authors who have proposed modifications to the model and to the algorithm; among which are worth mentioning: Lee and Plenert (1993) [18], Hsu and Chung (1998) [29], Onwubolu (2001) [23], Onwubolu and Mutungi (2001) [25], Vasant (2004) [29], Mishra, Prakash, Tiwari, Shankar, and Chan (2005) [24], Bhattacharya and Vasant (2007) [30] and Linhares (2009) [26].

As noted at the beginning, works about the optimal product mix under the Theory of Constraints require different working conditions in the textile industry. So, an application of some of the findings made by the authors to the problem under study must be reviewed and adapted to the particularities of the production of textiles.

It is also worth noting that the theory of constraints in general and its method of determining the optimal product mix in particular are aimed at the operation of production rather than the strategic direction of the company. Thus, its philosophy, although it has been extended to the entire business context, does not correspond at all with the objective of directing the marketing policy, but to improve productivity in the operation plant.

3. Numerical Example

The following tables summarize an example of the application of the algorithm to a case in the textile industry.

**Table 1. Product information**

<table>
<thead>
<tr>
<th>Product</th>
<th>V</th>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Looms group</td>
<td>B</td>
<td>A</td>
<td>C</td>
<td>E</td>
<td>D</td>
</tr>
<tr>
<td>Fill Yarns / cm.</td>
<td>10.4</td>
<td>24.4</td>
<td>15</td>
<td>15</td>
<td>5.83</td>
</tr>
<tr>
<td>Actual Efficiency (%)</td>
<td>40.9</td>
<td>48.3</td>
<td>57.1</td>
<td>57.1</td>
<td>45.7</td>
</tr>
<tr>
<td>Outputs</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1st Raw Material (code)</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>x100 kg 1st Raw Material / roll</td>
<td>1.2</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td>3.3</td>
</tr>
<tr>
<td>2nd Raw Material (code)</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x100 kg 2nd Raw Material / roll</td>
<td>0.4</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Sales (rolls)</td>
<td>8</td>
<td>89</td>
<td>18</td>
<td>25</td>
<td>59</td>
</tr>
<tr>
<td>Profit / roll</td>
<td>19.8</td>
<td>30.5</td>
<td>11</td>
<td>21.5</td>
<td>20.5</td>
</tr>
<tr>
<td>Roll length (mts)</td>
<td>200</td>
<td>300</td>
<td>300</td>
<td>700</td>
<td>1,025</td>
</tr>
</tbody>
</table>

Source: Authors’ contribution
Table 1 summarizes the technical information of the products, including the group of looms (to compare to Table 2), the density of the fabric in wefts per centimeter (important for speed), the actual efficiency of the product in the machines, the simultaneous product outputs on the looms, raw materials (to compare with Table 3), amount of raw material in a roll of fabric, market restrictions in rolls, the profit per unit and the length of the fabric roll in meters.

Table 2 describes the group of looms, identifying both the looms included in each group and its total production capacity in millions of wefts.

Table 3 summarizes the availability of Raw Material. It is an important constraint in industries such as technical textiles where suppliers are in distant countries and immediate availability is required to improve response times.

The exact mathematical model would be as follows:

Max $W = 0.024676(Y_{11}+Y_{12}+Y_{14}+Y_{15}+Y_{16}) + 0.010833(Y_{21}) + 0.006430(Y_{31}+Y_{33}+Y_{38}) + 0.005373(Y_{41}+Y_{43}+Y_{47}+Y_{48}+Y_{49}) + 0.009277(Y_{53}+Y_{510})$

Subject to:

$Y_{11}+Y_{21}+Y_{33}+Y_{41} \leq 43,200$

$Y_{12} \leq 43,200$

$Y_{33}+Y_{43}+Y_{53} \leq 43,200$

$Y_{14} \leq 43,200$

$Y_{15} \leq 43,200$

$Y_{16} \leq 43,200$

$Y_{47} \leq 43,200$

$Y_{48} \leq 43,200$

$Y_{49} \leq 43,200$

$Y_{510} \leq 43,200$

$0.000511(Y_{11}+Y_{12}+Y_{14}+Y_{15}+Y_{16}) \leq 8$

$0.000172(Y_{21}) \leq 89$

$0.000668(Y_{33}+Y_{38}) \leq 18$

$0.000285(Y_{41}+Y_{43}+Y_{47}+Y_{48}+Y_{49}) \leq 25$

$0.009277(Y_{53}+Y_{510}) \leq 59$

$0.61306(Y_{11}+Y_{12}+Y_{14}+Y_{15}+Y_{16}) \leq 20,000$

$0.17153(Y_{21}) \leq 20,000$

Following the procedure introduced in this paper, where products are sorted by their contribution margin per unit of time in the bottleneck resource and machines are assigned in order of flexibility index ($FI$) calculated as explained in the Methodology section, as is shown in Table 4. Shadowed cells are the minor group of looms for each loom. The final solution of the example is presented in Table 5.
The flexibility index introduced in this paper is a measure of the machine capacity to process different products that must be used to reserve the most flexible resources for specialized products without sacrificing productivity.

It is suggested to calculate the throughput as the difference between the selling price and the cost of processing instead of using Goldratt's formula which takes into account only the cost of raw materials.

Although the heuristic of the Theory of Constraints does not get close enough to the global optimum of the optimal product mix problem, it provides an overview of the classification of the products in ascendant order of profits. This overview might be useful in strategic market direction and capacity expansion projects.

When considering the details of scheduling in the plant, TOC heuristics are more specific and operational than models of linear programming. In addition, TOC heuristics are easy to implement since they do not involve complex calculations.

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